Computer Arithmetic - Fall 2001
Assignment No. 2
Course homepage: `http://hyde.eng.tau.ac.il/Arith01/arith.html`

Guidelines.

1. Submission deadline is December 20th - before the beginning of the lecture.
2. Do not submit in pairs. You may discuss the homework with your colleagues, however, you must phrase and write it by yourself.

Questions:

1. Why is two's complement representation defined by:

$$[x[n-1:0]] = -2^{n-1} \cdot x[n-1] + \langle x[n-2:0] \rangle ?$$

Let $\tau : [-1,1) \longleftarrow [0,2)$ denote the function defined by

$$\tau(x) = \begin{cases} x & \text{if } x \in [0,1) \\ 2+x & \text{if } x \in [-1,0). \end{cases}$$

It is useful to think of τ as a "two's complement encoding function". Note that $\tau(x) = x(\text{mod} 2)$.

   (a) Define the reverse function τ^{-1}. It is useful to think of τ^{-1} as a function that decodes the value represented by a "two's complement number".

   (b) Consider a radix polynomial $x[-1:p-1]$ that represents the value

$$\langle x[-1:p-1] \rangle = \sum_{i=-1}^{p-1} x[i] \cdot 2^{-i}.$$

   Prove that $\tau^{-1}(\langle x[-1:p-1] \rangle) = [x[-1:p-1]]$.

2. Computation of absolute value of a difference. Let $a[-1:p-1], b[-1:p-1] \in \{0,1\}^p$ denote two binary strings that are binary representations of fractions in the range $[1,2)$.

   (a) Design the fastest circuit you can that computes the binary representation of the absolute value of $\langle a \rangle - \langle b \rangle$.

   (b) Compare the delay of your design with the delay of a binary adder.

   (c) Prove the correctness of your design.

3. Priority Encoder (or Leading Zeros Counter). A Priority Encoder is a combinational circuit defined as follows.

   Input: A non-zero binary string $a[1:n]$ (where n is a power of 2, i.e., $n = 2^k$).

   Output: A binary string $b = [k-1:0]$.

Functionality: The output b satisfies $a[1 : \langle b \rangle] = 0^{\langle b \rangle - 1} \cdot 1$.

Note that to make the definition "easier" the bits of a are indexed from left to right with increasing indexes. This is just an issue of notation.

(a) Design a priority encoder with logarithmic depth and linear cost.

(b) Prove the correctness of your design.

4. Normalization shift. A radix polynomial $x[-1 : p-1]$ is *normalized* if $\langle x[-1 : p-1] \rangle \in [1, 2)$.

   (a) Design the fastest circuit you can for computing the normalized absolute value of a difference of two binary fractions. Formally, the input is two radix polynomials $a[-1 : p-1], b[-1 : p-1] \in \{0, 1\}^p$, where $a \neq b$. The goal is to output $d \in \{0, 1\}^p$ such that:

      i. d is normalized.

      ii. There exists a non negative integer k such that $2^k \cdot \langle d \rangle = abs(\langle a \rangle - \langle b \rangle)$. ($k$ is often referred to as the *normalization shift amount.*)

   (b) Compare the delay of your design with the delay of a priority encoder and a binary adder.

   (c) Prove the correctness of your design.