Computer Arithmetic - Fall 2001
Assignment No. 3 (take home exam)
Course homepage: `http://hyde.eng.tau.ac.il/Arith01/arith.html`

Guidelines.

1. Submission deadline is March 10.
2. You may submit in pairs.
3. Submission must be typed not handwritten.
4. You may do a project instead provided that you submit a progress report by February 21.

Questions:

1. (25 %) Let $A[0 : p-1], B[0 : p-1]$ denote two normalized significands (namely, $\langle A\rangle, \langle B\rangle \in [1, 2)$). Let $P[-1 : 2(p-1)], Q[-1 : 2(p-1)]$ satisfy

$$\langle P\rangle + \langle Q\rangle = \langle A\rangle \cdot \langle B\rangle + \textit{injection}.$$

Consider the following definitions:

$$\begin{aligned}(P'[-1 : (p-1)], Q'[-1 : (p-2)]) &\leftarrow \text{Half-Adder}(P[-1 : (p-1)], Q[-1 : (p-1)]).\\ \langle Y[-1 : (p-2)]\rangle &\leftarrow \langle P'[-1 : (p-2)]\rangle + \langle Q'[-1 : (p-2)]\rangle\\ \langle Y'[-1 : (p-2)]\rangle &\leftarrow \langle P'[-1 : (p-2)]\rangle + \langle Q'[-1 : (p-2)]\rangle + 2^{-(p-2)}.\end{aligned}$$

Prove the following:

(a) If $\langle A\rangle \cdot \langle B\rangle \geq 2$ and $Y[-1] = 0$, then $round(\langle A\rangle \cdot \langle B\rangle) = 2$.

(b) If $\langle A\rangle \cdot \langle B\rangle < 2$ and $Y[-1] = 1$, then $round(\langle A\rangle \cdot \langle B\rangle) = 2$.

2. (75%) Analyze the errors in the division algorithm of AMD's K7 FPU (consider only double precision).

(a) Compute an upper bound on the precision of the multiplications if you keep the precision of the initial approximation unchanged

(b) Compute an upper bound on the precision of the initial approximation if you keep the precision of the multiplications unchanged.