Network Algorithms - Fall 2002
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Deadline: Wednesday 16/10/02. Due to short notice, volunteers will be asked to answer rather than a random sample.

[CCPS] refers to the book "Combinatorial Optimization" by Cook, Cunningham, Pulleyblank, and Schrijver.

Handout #1: Maximum Flow

1. Prove Proposition 3.2 in [CCPS]: Every (r, s)-flow of non-negative value is the sum of at most m flows, each of which is a path flow or a circuit flow. (m - number of edges in the directed graph).

2. Prove Theorem 3.7 in [CCPS]: If the edge capacities are integral and there exists a maximum flow, then there exists a maximum flow that it integral. (A function is *integral* if its range is contained in the set of integers.)

3. Answer exercise 3.3 in [CCPS]: Prove directly (without relying on the max flow - min cut theorem) that there is no maximum flow if and only if there is an (r, s)-dipath (a directed path), each of whose arcs has infinite capacity.

4. Answer exercise 3.19 in [CCPS]: Consider a matrix $A = \{a_{ij}\}$. Prove that the maximum number of non-zero entries, no two in the same row or column, equals the minimum number of rows or columns that contain all the non-zero entries.

5. Prove Hall's Marriage Theorem: Let $G = (X \cup Y, E)$ denote a bipartite graph. For every $A \subseteq X$, we denote the set of neighbors of vertices of A by $\Gamma(A)$. Namely,
$$\Gamma(A) = \{y \in Y : \exists x \in A \text{ such that } xy \in E\}.$$
A matching M is *complete* if $|M| = |X|$.

   Prove that G has a complete matching if and only if $|\Gamma(A)| \geq |A|$, for every $A \subseteq X$.