Network Algorithms - Fall 2002
Dr. Guy Even
http://hyde.eng.tau.ac.il/CO02/

Handout #6: Graphical representations of linear programs

Deadline: Monday 16/12/02.

1. Consider the problem:

$$\begin{array}{ll} \text{minimize} & -x_1 - x_2 \\ \text{subject to} & x_1 + 2x_2 \leq 3 \\ & 2x_1 + x_2 \leq 3 \\ & x_1, x_2 \geq 0. \end{array}$$

   (a) Draw the polyhedron $P \subseteq \mathbb{R}^2$ defined by the constraints.
   (b) Fix a value z and draw the hyperplane $\mathbf{c}'\mathbf{x} = z$, where $\mathbf{c} = (-1, -1)$ and $\mathbf{x} = (x_1, x_2)$. Note that this hyperplane is perpendicular to $\mathbf{c}$.
   (c) Find the smallest value z, such that the hyperplane $\mathbf{c}' \cdot \mathbf{x} = z$ intersects the polyhedron P. Note that, for the smallest such z, the hyperplane intersects z at a corner.

2. Consider the polyhedron $P \subseteq \mathbb{R}^3$ defined by the constraints:

$$0 \leq x_i \leq 1 \quad \text{for every } i = 1, 2, 3.$$

   This polyhedron is called the *unit cube*.

   Consider the vector $\mathbf{c} = (-1, -1, -1)$. Find the point $\mathbf{x} \in P$ that minimizes $\mathbf{c}' \cdot \mathbf{x}$. Is that point again a corner?

3. Consider the polyhedron $P \subseteq \mathbb{R}^2$ defined by the constraints:

$$\begin{aligned} -x_1 + x_2 &\leq 1 \\ x_1, x_2 &\geq 0 \end{aligned}$$

   (a) What is the solution of minimize $\mathbf{c}' \cdot \mathbf{x}$ for $\mathbf{c} = (1, 1)$?
   (b) What is the solution of minimize $\mathbf{c}' \cdot \mathbf{x}$ for $\mathbf{c} = (1, 0)$?
   (c) What is the solution of minimize $\mathbf{c}' \cdot \mathbf{x}$ for $\mathbf{c} = (0, 1)$?
   (d) What is the solution of minimize $\mathbf{c}' \cdot \mathbf{x}$ for $\mathbf{c} = (-1, -1)$?
   (e) Suppose we impose an aditional constaraint of the form $x_1 + x_2 \leq 2$. What can you say about the new polyhedron?