Network Algorithms - Fall 2002

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Handout #9: Duality

Deadline: Monday 13/01/03.

1. Write linear programs and their dual for the following problems. Try to interpret the combinatorial meaning of the dual programs.

   - max flow,
   - min cut,
   - shortest path from s to t in an undirected graph,
   - min cost flow,
   - min weight perfect matching.

   Remark: Consider two versions: one with a polynomial number of constraints and one with exponentially many constraints (e.g., max-flow with one constraint per st-path).

2. Consider the linear program Π defined by:

   $$\min\{\mathbf{c}' \cdot \mathbf{x} \mid \mathbf{A}\mathbf{x} = \mathbf{b}, \mathbf{x} \geq \mathbf{0}\}.$$

   Assume that Π is feasible. Assume also that $rank(\mathbf{A}) < m$. Specifically, assume that $\mathbf{a}_m = \sum_{i=1}^{m-1} \gamma_i \cdot \mathbf{a}_i$. Moreover, assume that $\mathbf{b}_m = \sum_{i=1}^{m-1} \gamma_i \cdot \mathbf{b}_i$ (hence the last constraint is redundant).

   Let Π' denote the linear program obtained by omitting the mth redundant constraint $\mathbf{a}'_m \cdot \mathbf{x} = \mathbf{b}_m$ from Π.

   Let Δ and Δ' denote the duals of Π and Π', respectively. Prove that Δ and Δ' are equivalent.

3. (extra bonus!) Present a demo of the Simplex Algorithm (question 1 from last set). Make sure you fully prepare the demo in advance and can explain the whole process.