Introduction to Digital Computers - Spring 2001

Assignment No. 4

Course homepage: http://hyde.eng.tau.ac.il/Digital_Computers01/

Firm Deadline: April 19th - before the beginning of the lecture

1. Assume the fanout of nets is limited by 2 and we use 1-buffers when larger fanouts are required.

   (a) Make the necessary changes to the designs of $CSA(n)$, $ST-ADD(n)$ and $CLA(n)$ so that fanout limitations are not violated.

   (b) Analyze the cost and delay for each adder (with fanout limitations) as a function of n. Write the recurrence and solve the asymptotics.

   (c) Calculate (numerically) the cost and delay for each adder, with *and* without fanout limitations, for the Motorola "technology" for $n = 4, 8, 16, 32, 64, 128$. Assume the cost of a 1-buffer equals 1 and its delay equals 1.

   (d) For each category (n, with/without fanout limitations, cost and delay), which adder is the best?

2. What does a two's complement adder output when an overflow occurs? Let:

   - $a[n-1:0], b[n-1:0] \in \{0,1\}^n$; and
   - $z = [a[n-1:0]] + [b[n-1:0]]$.

   Suppose that

   $$\langle c[n] \cdot s[n-1:0] \rangle = \langle a[n-1:0] \rangle + \langle b[n-1:0] \rangle.$$

   Prove that:

   (a)

   $$[s[n-1:0]] = z \pmod{2^n}$$

   (b) Suppose that $z \notin T_n$ and that $z > 0$. Write the smallest interval that z belongs to. Write a closed formula for $[s[n-1:0]]$. What is the interval that $[s[n-1:0]]$ belongs to?

   (c) Suppose that $z \notin T_n$ and that $z < 0$. Write the smallest interval that z belongs to. Write a closed formula for $[s[n-1:0]]$. What is the interval that $[s[n-1:0]]$ belongs to?

   (d) Write and execute a C program that demonstrates your answers. Clue: a and b are added as unsigned and stored in an unsigned variable s. Print a, b, and s as signed integers. (You may use another programming language if you like.)

3. Unsigned subtraction. Let:

   - $a[n-1:0], b[n-1:0] \in \{0,1\}^n$; and
   - $z = \langle a[n-1:0] \rangle - \langle b[n-1:0] \rangle$

   Suppose that

   $$\langle c[n] \cdot s[n-1:0] \rangle = \langle a[n-1:0] \rangle + \langle \overline{b[n-1:0]} \rangle + 1.$$

   Answer the same questions asked for addition for this case.

4. Let $\langle c[n] \cdot s[n-1:0] \rangle = \langle a[n-1:0] \rangle + \langle b[n-1:0] \rangle$. We know that an overflow occurs in two's complement addition if and only if $c[n] \neq c[n-1]$. Prove that

$$c[n] \neq c[n-1] \text{ iff } (a[n-1] = b[n-1]) \text{ and } (a[n-1] \neq s[n-1])$$