Introduction to Digital Computers

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This course is about designing digital circuits.

- What is a digital circuit?

- What are digital circuits built of?

- What do digital circuits do?

- How are digital circuits built?

Syntax vs. Semantics

- Syntax - rules for building large objects from smaller objects. How does one design large circuits from smaller ones?

- Semantics -
  - What does a circuit do? an adder "adds" numbers...
  - What is the meaning of a circuit? an inverter outputs a value "opposite" to the input value...
  - What is the behavior of a circuit?

Analog signals

An analog signal is a real function

$$f : \mathbb{R} \longrightarrow \mathbb{R}$$

that describes the voltage of a given point in a circuit as a function of the time.

Assumption: wires have zero resistance, zero capacity. Signals propagate through wires immediately.

$\Rightarrow$ voltage along a wire is identical at all times.

Since a signal describes the voltage, we also assume that a signal is a continuous function.

Digital signals

A digital signal is a function

$$g : \mathbb{R} \longrightarrow \{0, 1, \text{non-logical}\}$$

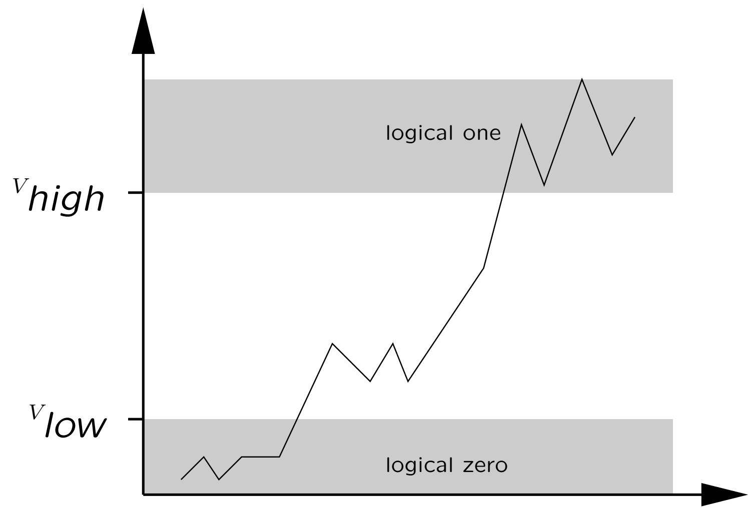
The value of a digital signal describes the logical value carried along a wire as a function of time.

- Zero & one : logical values.
- Non-logical: indicates that the signal is not digital.

Interpreting analog signals as digital signals

How does one interpret an analog signal as a digital signal? Two voltage thresholds are defined: $V_{\mathit{low}} < V_{\mathit{high}}$. Consider an analog signal $f(t)$. The digital signal $\mathit{dig}(f(t))$ is defined as follows.

$$\mathit{dig}(f(t)) \triangleq \begin{cases} 0 & \text{if } f(t) < V_{\mathit{low}} \\ 1 & \text{if } f(t) > V_{\mathit{high}} \\ \text{non-logical} & \text{otherwise.} \end{cases}$$


logical one
Vhigh
Vlow
logical zero

Question:

Define an inverter.

Warning! useless definition!

The digital interpretation of analog signals defined previously is useless.

Why?

Before we can answer that we need to discuss transfer functions...

Transfer functions

The voltage at an output of a gate depends on the voltages of the inputs of the gate. This dependence is called the transfer function.

Example: an inverter with an input x and an output y. The value of the signal $y(t)$ at time t is a function of the signal x in the interval $(-\infty, t]$.

Static transfer function: if the input $x(t)$ is stable for a sufficiently long period of time and equals x_0, then the output $y(t)$ stabilizes on a value y_0 that is a function of x_0.

(If this were not the case, then a small device with very few transistors could be used to "store" the whole "history".)

Static transfer function

We formalize the definition of a static transfer function of a gate G with one input x and one output y in the following definition.

Definition 1 *A function* $f : \mathbb{R} \to \mathbb{R}$ *is a* static transfer function *of a gate* G *if there exists a* $\Delta > 0$, *such that, for every* x_0 *and every* t_0, *if* $x(t) = x_0$ *for every* $t \in [t_0 - \Delta, t_0]$, *then* $y(t_0) = f(x_0)$.

Static transfer function - remark 1

Domains and ranges of static transfer functions are usually bounded (say $[0,5]$ volts).

Designers of digital circuits are mostly interested in the values of transfer functions only over the bounded domains that correspond to logical zero and logical one values. The reason is that Δ might be prohibitively big when x_0 corresponds to a non-logical value.

Static transfer function - remark 2

To make the definition useful, one should allow perturbations of $x(t)$ during the interval $[t_0 - \Delta, t_0]$. For example:

$$\forall \delta > 0 \ \exists \epsilon > 0 : |x(t) - x_0| \leq \epsilon \Rightarrow |y(t_0) - f(x_0)| \leq \delta.$$

Static transfer function - remark 3

The constant Δ does not depend on x_0 (otherwise we would not have referred to it as a constant!).

This constant is called the propagation delay of the gate G and is perhaps the most important characteristic of a gate.

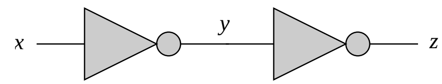
An attempt to define an inverter

Definition 2 *A gate G with a single input x and a single output y is an inverter if its static transfer function $f(z)$ satisfies the following the following two conditions:*

1. *If $z < V_{\text{low}}$, then $f(z) > V_{\text{high}}$.*

2. *If $z > V_{\text{high}}$, then $f(z) < V_{\text{low}}$.*

Noise



Assume that:

- $x > V_{high}$, so $dig(x) = 1$,
- $y = V_{low} - \epsilon$, for a very small $\epsilon > 0$.
- $\Rightarrow dig(z) = 1$.

Everything seems fine... But what if noise $n_y(t)$ is added to the wire y? (what is noise?!)

If $n(t) > \epsilon$, then *dig*(y) = non-logical, and can't deduce that $dig(z) = 1$.

Noise margins