Hardware Algorithms: circuits & networks
Problem Set 2

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1. Consider n identical finite state machines (soldiers) arranged in a line. At time t = 0, every soldier is initialized to the same state, except for the soldier on the far left (the general). Every soldier (including the general) sends its output to its neighbors.The initial state is stable in the sense that if in cycle t a soldier A and its neighbors are in the initial state, then A remains in the initial state in cycle t+1. In addition there is another state called the "sync" state. Synchronization is achieved after T cycles if in cycle T all soldiers enter state sync for the first time. Implement each soldier by a finite state machine (the number of states and size of alphabet may not depend on n) so that synchronization is achieved as soon as possible.
(Hint: you may assume that $n = 2^k$ or $n = 2^k - \ell$.)

2. Design the following combinational circuit:

   (a) Input: $x \in \{0,1\}^n$
   (b) Output: $y \in \{0,1\}^{\lg n}$
   (c) Functionality: $y = HammingWeight(x)$ (the Hamming weight of x is the number of nonzero components in x).

   The propagation delay of your design should be $O(\log n)$.

   (Hint: Try to analyze the depth of a tree of adders - and show that it is too deep.)

3. Reduce the following problem to PPC:

   (a) Input: $\{(i, y_i)\}_{i=1}^{n}$points in $\mathbb{R}$.
   (b) Functionality: $Convex_i \in \{0,1\}$ where $Convex_i = 1$ iff the piecewise-linear graph obtained by connecting the points $\{(k, y_k)\}_{k=1}^{i}$ is convex.

4. You are given a representation of F and a gate that computes an associative operator $* : F \times F \to F$. Design a PPC circuit that solves the following problem using a **single** $PPC(n)$ circuit:

   (a) Input: $\vec{x} \in F^n$ and $\vec{b} \in \{0,1\}^n$.
   (b) Output: $\vec{y} \in F^n$ such that
   $$y_i = x_i * x_{i-1} * \cdots * x_{j(i)},$$
   where $j(i) = \max\{j : b_j = 1 \text{ and } j < i\}$. (We define the maximum of an empty set to be 0.)

5. Compute $A + B$ and $A + B + 1$ using the $PPC - ADDER$ signals $\{\Pi_i\}$.