Hardware Algorithms: circuits & networks
Problem Set 4
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1. Consider the odd-even transposition algorithm. Give an example with n inputs for which $n-1$ levels do not suffice to sort the inputs.

2. Prove the 0-1 Principal for merging and bitonic merging.

   Hint: use the follow lemma. A vector v is bi-sorted/bitonic iff $f_t(v)$ is bi-sorted/bitonic for every threshold function $f_t : R \to \{0,1\}$ defined by $f_t(x) = 1 \Leftrightarrow x \geq t$.

3. Consider a variation of the odd-even merger in which the inputs A, B are connected to the two smaller mergers as follows:

   - top merger is input $odd(A)$ and $even(B)$, and
   - bottom merger is input $even(A)$ and $odd(B)$.

   (a) Suggest a completion of the circuit using a column of comparison boxes fed by the outputs of the top and bottom mergers.

   (b) Prove the correctness of the obtained merging circuit.

4. Bitonic merging. Given a bitonic vector $\{x_i\}_{i=1}^{2n}$, for $i = 1 \ldots n$, let

$$a_i = \min\{x_i, x_{i+n}\}$$
$$b_i = \max\{x_i, x_{i+n}\}.$$

   (a) Prove that either:

   - $\{a_i\}_{i=1}^{n} = 0^n$ and $\{b_i\}_{i=1}^{n}$ is bitonic, or
   - $\{b_i\}_{i=1}^{n} = 1^n$ and $\{a_i\}_{i=1}^{n}$ is bitonic.

   (b) Use the above claim to prove the correctness of Batcher's bitonic merging circuit.