Multi-Hop Routing and Scheduling in Wireless Networks in the SINR Model

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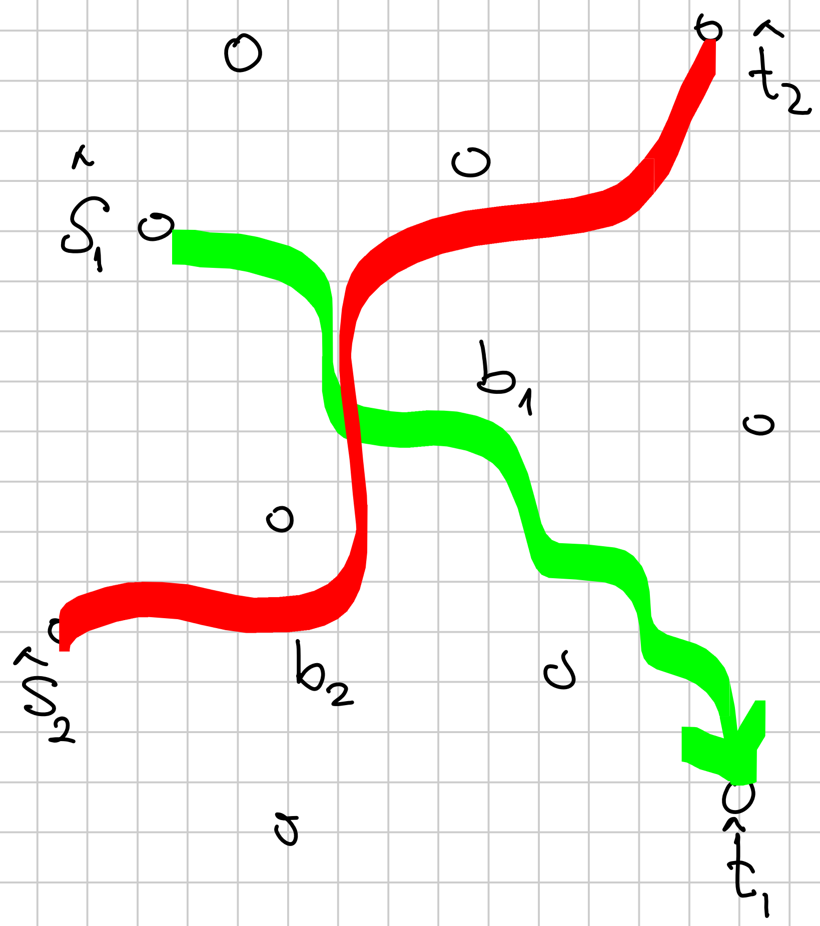
Problem: Throughput Maximization

1) n nodes in the plane
2) k requests $R_i = (\hat{s}_i, \hat{t}_i, b_i)$

want: schedule packets in network for the requests.



* packets may travel along different paths
* multi-hop: solve routing
* interference model: SINR

Spec: Output of Algorithm

Output: multi-commodity flow f & schedule $\{L_t\}_{t=1}^{T}$

multi-comm flow: $f = (f_1, \dots, f_K)$, where f_i for request R_i

Schedule: Seq. of T SINR-feas sets of links $\{L_t\}_{t=1}^{T}$.

Schedule supports flow:

$$\forall e: \quad \frac{f(e)}{c(e)} \le \frac{1}{T} \left| \{ t : e \in L_t \} \right|$$

Goals: $\max \sum_{i=1}^{T} \min \left\{ 1, \frac{|f_i|}{b_i} \right\}$ (throughput)

$\max \min_i \frac{|f_i|}{b_i}$ (max-min throughput)

offline: 1) queue management. 2) compare to shortest schedule.

Results

who	linear powers	given powers	powers $\in [P_{MIN}, P_{MAX}]$
Chafekar et al. Infcom 2008	$\log \Delta$ (also uniform power)	$\log \Gamma \cdot \log \Delta$	
Wan et al. Infocom 2011*	$\log n$ (also unlimited)		$\log \Gamma \cdot \log n$
We	$\log n$	$\log n \cdot (\log \Gamma + \log \Delta)$	$(\log n + \log\log \Gamma) \cdot (\log \Gamma + \log \Delta)$

$$\Gamma \stackrel{\Delta}{=} \frac{P_{max}}{P_{min}} \qquad \Delta = \frac{d_{max}}{d_{min}}$$

* Wan: "only of theoretical interest and quite infeasible practically"

Assumptions

1) Same SINR threshold for all the links.

2) all links use a single modecode.

3) uniform fixed packet length.

$\Rightarrow$ each link can deliver one packet per slot.

4) links bounded away from SINR threshold

$$(*) \qquad \frac{S_e}{N} \geq (1+\varepsilon)\cdot\beta$$

offline: is (*) a technicality or a "real" issue

The Plan

1) formulate an LP-relaxation

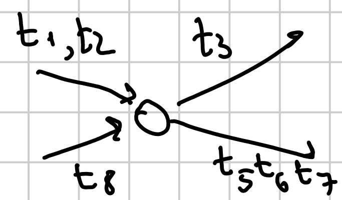
Solution: $f = (f_1, \ldots, f_k)$ that serves $(R_1, \ldots, R_k)$

2) compute an SINR-feasible schedule $\{L_t\}_{t=1}^{T}$

that supports (a fraction of) f.

offline : distributed vs. centralized

given $(f, \{L_t\}_{t=1}^{T})$, control is local



LP-Relaxation

1) multi-comm flow $f = (f_1, \dots, f_k)$.

f_i is a flow for request $R_i = (\hat{s}_i, \hat{t}_i, b_i)$

2) But, must address interferences. HOW?

graph model: $$\frac{f(e)}{c(e)} + \sum_{\substack{e' \text{ interfers} \\ \text{with } e}} \frac{f(e')}{c(e')} \leq 1$$

* pessimistic * amenable to "greedy" coloring

SINR model?

call affectance for help!

Affectance

$$\frac{S_e}{N + I_e} \geq \beta \quad \Longrightarrow \quad I_{max}(e) = \frac{S_e}{\beta} - N$$

$$a_{e'}(e) = \frac{S_{e'e}}{I_{max}(e)}$$

Claim: a subset L of links is SINR feas.

$$\Longleftrightarrow \quad \forall e \in L: \quad a_L(e) \leq 1$$

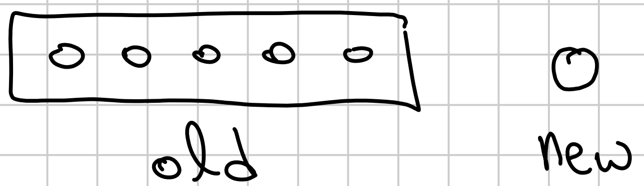
LP constraint? $\quad f(e) + \sum_{e'} a_{e'}(e) \cdot f(e') \leq 1$

1) Not a relaxation! if $a_{e'}(e) \gg 1$ then

$f(e) = f(e') = \frac{1}{2}$ is not feas.

Solution: $\quad \bar{a}_{e'}(e) \triangleq \min\{ a_{e'}(e), 1 \}$

2) How to schedule? $\quad \bar{a}_{e'}(e) \neq \bar{a}_e(e')$



$a_{OLD}(new) \leq 1$

but $a_{new}(e) > 1$ for $e \in old$

Buckets

Classify links by reception power:

$$B_i = \{ e : 2^i \cdot S_{min} \leq S_e < 2^{i+1} \cdot S_{min} \}$$

linear powers: one bucket.

LP relaxation: interference constraint

$$\forall i: \forall e \in B_i \quad f(e) + \sum_{\{e' \in B_i : d_{e'} \geq d_e\}} (\bar{a}_{e'}(e) + \bar{a}_e(e')) \cdot f(e') \leq 1$$

remark: Sum over $d_{e'} \geq d_e$ does not complicate scheduling.

Why is a relaxation? Suppose $L \subseteq B_i$ is SINR feas.

Thm 1: $\forall e \in B_i \quad \sum_{e' \in L} \bar{a}_{e'}(e) = O(1)$

Thm 2 [K11]: $\forall e \in B_i \quad \sum_{e' \in L : d_{e'} \geq d_e} \bar{a}_e(e') = O(1)$

relaxation: use averaging argument...

Greedy Coloring

Given flow f (solution of LP), computes a schedule $\{L_t\}_{t=1}^{T}$ that supports f/λ (λ is constant).

Each L_t satisfies: $\forall e \in L \quad \sum_{e' \in L:\, d_{e'} \geq d_e} \left(\bar{a}_{e'}(e) + \bar{a}_e(e')\right) \leq 1$.

How? fix T large enough

1) Scan links in descending length order

2) allocate $\lfloor T \cdot f(e) \rfloor$ time slots to e. (first fit)

always succeeds because $f(e) + \sum \left(\bar{a}_{e'}(e) + \bar{a}_e(e')\right) f(e') \leq 1$

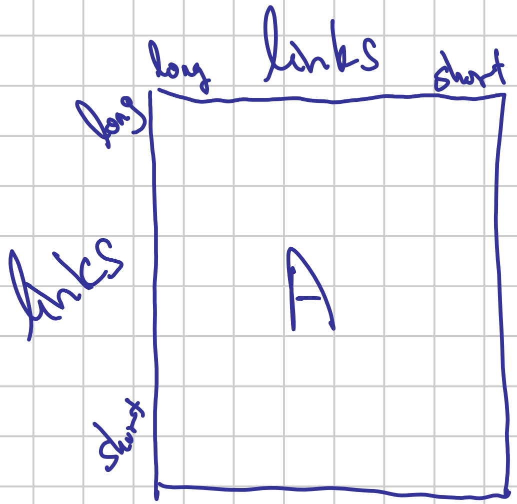
dispersion (making schedule SINR feas)

1) Refine each L_t into $O(\log n)$ subsets $\{L_{t,i}\}$.

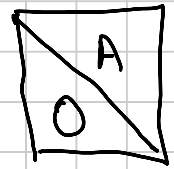
$$\forall e \in L_{t,i}: \quad \bar{a}_{L_{t,i}}(e) \leq 3$$

2) Further refine to obtain SINR-feas [HW09].

$$\sum_{e' : d(e') \geq d(e)} \left(\bar{a}_{e'}(e) + \bar{a}_e(e') \right) \leq 1 \implies a_{L_{t,i}}(e) \leq 3$$

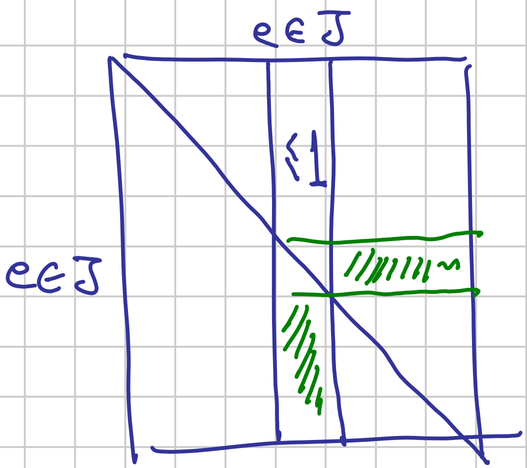


$A(e', e) \triangleq \bar{a}_{e'}(e) + \bar{a}_e(e')$

we know: in  every column has weight ≤ 1.

$\overset{\text{Markov}}{\implies}$ at least half rows have weight ≤ 2

denote those by J.



$\implies$ weight of col $e \in J$ in A is $\leq 1 + 2 = 3$.

Each J contains half the remaining links

So after $(\log n)$ iteration, we stop.

More than one bucket?

- deal with each bucket separately.
- concatenate schedules.
- decreases throughput by # buckets.

$$\# \text{buckets} \leq \log\left(\frac{S_{max}}{S_{min}}\right) = \log\left(\frac{P_{max}}{P_{min}} \cdot \left(\frac{d_{max}}{d_{min}}\right)^{\alpha}\right)$$

$$= \log \Gamma + \alpha \cdot \log \Delta$$

$$\Rightarrow \quad O(\log n \cdot (\log \Gamma + \log \Delta)) \text{ approx.}$$

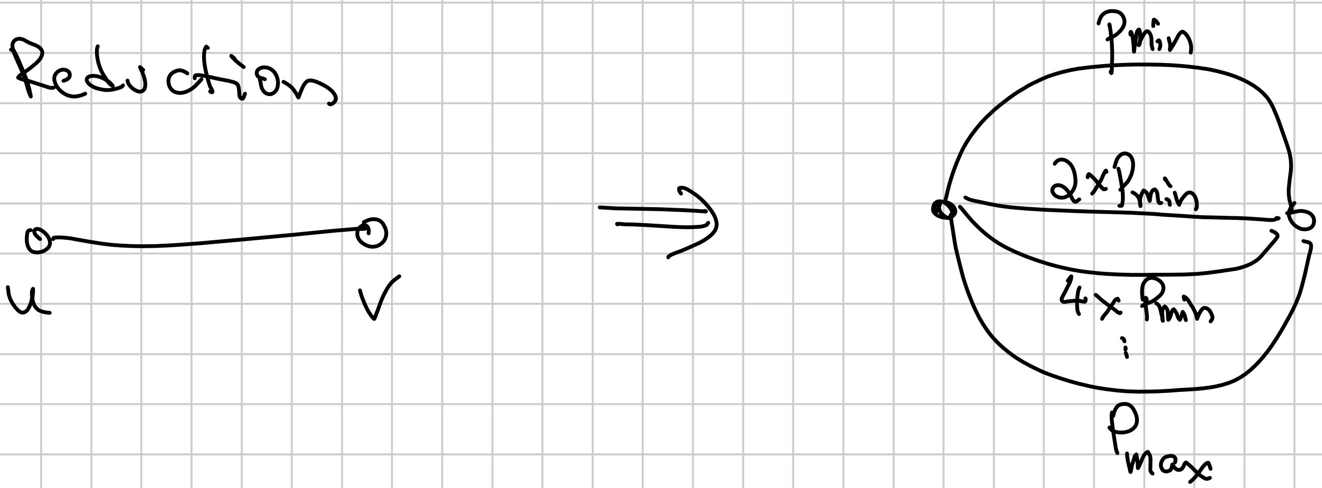
Limited Powers

each link e may transmit with power $P_e \in [P_{min}, P_{max}]$.

alg needs to compute powers.

link may change power between time slots.

Reduction



for simplicity, assume: $P_{max} = 2^k \cdot P_{min}$.

observation: Limiting opt to discrete powers reduces throughput by a constant.

$$\text{OPT} \longrightarrow \quad \forall e \in L_t : \quad P_e' \leftarrow \min \left\{ 2^i \cdot P_{min} \;\middle|\; \underset{\text{or } P_e = P_{max}}{2^i P_{min} > P_e} \right\}$$

Now L_t^{disc} is not SINR feas, but it is a