Pearls of Shimon Even

פנינים של אבן

graphs *algs* *P* *NP* *PSPACE* *crypto*

A talk in memory of my beloved father
The Academic College of Tel-Aviv Yaffo
June 3, 2004

Lots of great works

- Graph Algorithms: planarity testing, zero-one flows, connectivity, matching, dynamic algorithms.
- NP Completeness: timetables, integral multicommodity flows, ...
- PSPACE Completeness: Hex.
- Approximation Algorithms: vertex cover, local ratio.
- Cryptography: electronic wallet, digital signatures, signing contracts.
- Distributed Computation: synchronization, broadcast.
- Systolic Circuits: marked graphs, algorithms for retiming, Atrubin's multiplier.

Lots of great works

- Graph Algorithms: planarity testing, zero-one flows, connectivity, matching, dynamic algorithms.
- NP Completeness: timetables, integral multicommodity flows, ...
- PSPACE Completeness: Hex.
- Approximation Algorithms: vertex cover, local ratio.
- Cryptography: electronic wallet, digital signatures, signing contracts.
- Distributed Computation: synchronization, broadcast.
- Systolic Circuits: marked graphs, algorithms for retiming, Atrubin's multiplier.

Question: What should I talk about?

Layouts of graphs

- Wonderful topic

Layouts of graphs

- Wonderful topic
- Lots of pictures

Layouts of graphs

- Wonderful topic
- Lots of pictures
- Work on layouts spanned 40 years

Layouts of graphs

- Wonderful topic
- Lots of pictures
- Work on layouts spanned 40 years
- ...we even have a joint paper about it!

Graphs

- Consider a set of 5 people:

Graphs

- Consider a set of 5 people:

  Avi, Benny, Cindy, Danny, Ella.

Graphs

- Consider a set of 5 people:

  Avi, Benny, Cindy, Danny, Ella.

- Write down the pairs of people that know each other:

Graphs

- Consider a set of 5 people:

  Avi, Benny, Cindy, Danny, Ella.

- Write down the pairs of people that know each other:
  - Avi knows Benny: (A, B)

Graphs

- Consider a set of 5 people:

  Avi, Benny, Cindy, Danny, Ella.

- Write down the pairs of people that know each other:
  - Avi knows Benny: (A, B)
  - Benny knows Cindy: (B, C)

Graphs

- Consider a set of 5 people:

  Avi, Benny, Cindy, Danny, Ella.

- Write down the pairs of people that know each other:
  - Avi knows Benny: (A, B)
  - Benny knows Cindy: (B, C)
  - and also (C, D), (D, E), (E, A).

Graphs

- Consider a set of 5 people:

  Avi, Benny, Cindy, Danny, Ella.

- Write down the pairs of people that know each other:
  - Avi knows Benny: (A, B)
  - Benny knows Cindy: (B, C)
  - and also (C, D), (D, E), (E, A).
  - graph $=$ set of people and list of pairs of people.

Graphs

- Consider a set of 5 people:

  Avi, Benny, Cindy, Danny, Ella.

- Write down the pairs of people that know each other:
  - Avi knows Benny: (A, B)
  - Benny knows Cindy: (B, C)
  - and also (C, D), (D, E), (E, A).
  - graph $=$ set of people and list of pairs of people.
  - Other examples: cities & roads, electrical components & wires, etc.

Depiction of graphs

Consider the graph with

- People: Avi, Benny, Cindy, Danny, Ella
- Pairs of people: $(A, B), (B, C), (C, D), (D, E), (E, A)$.

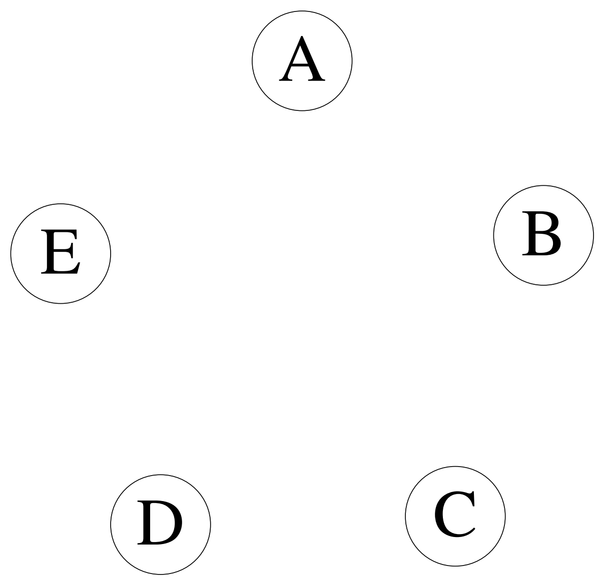
Good idea to draw it:

Depiction of graphs

Consider the graph with

- People: Avi, Benny, Cindy, Danny, Ella
- Pairs of people: $(A, B), (B, C), (C, D), (D, E), (E, A)$.

Good idea to draw it:

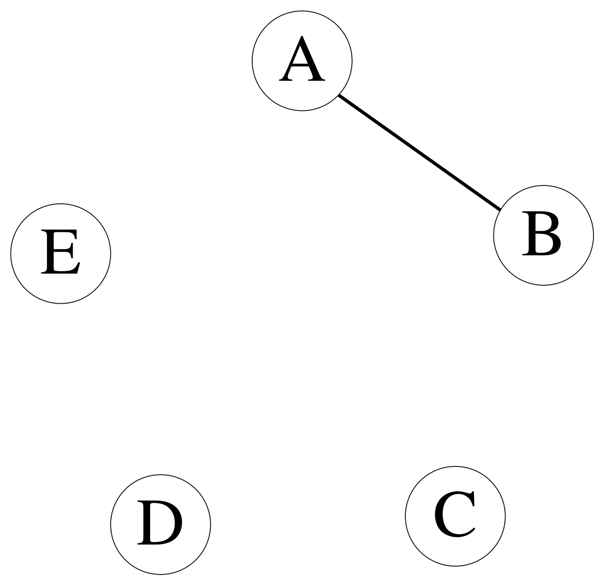


Depiction of graphs

Consider the graph with

- People: Avi, Benny, Cindy, Danny, Ella
- Pairs of people: $(A, B), (B, C), (C, D), (D, E), (E, A)$.

Good idea to draw it:

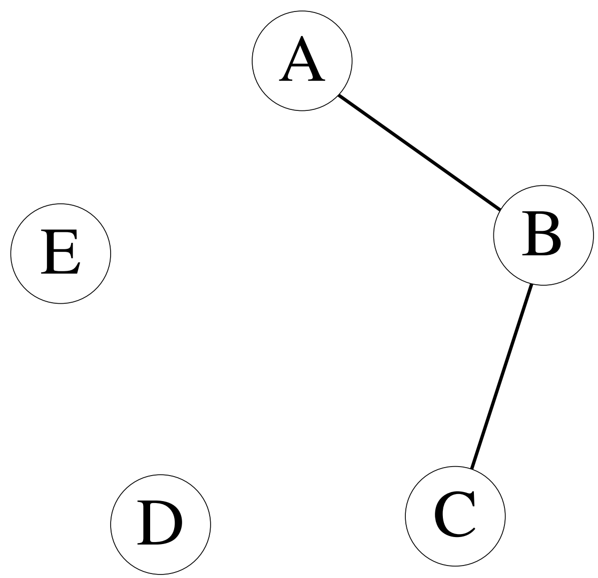


Depiction of graphs

Consider the graph with

- People: Avi, Benny, Cindy, Danny, Ella
- Pairs of people: $(A, B), (B, C), (C, D), (D, E), (E, A)$.

Good idea to draw it:

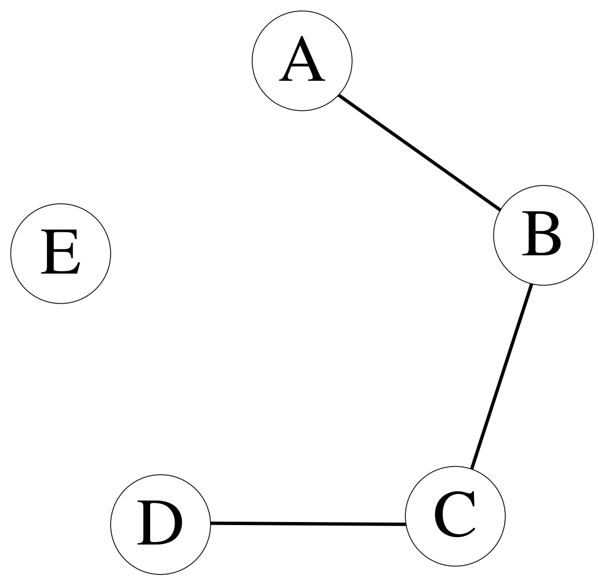


Depiction of graphs

Consider the graph with

- People: Avi, Benny, Cindy, Danny, Ella
- Pairs of people: $(A, B), (B, C), (C, D), (D, E), (E, A)$.

Good idea to draw it:

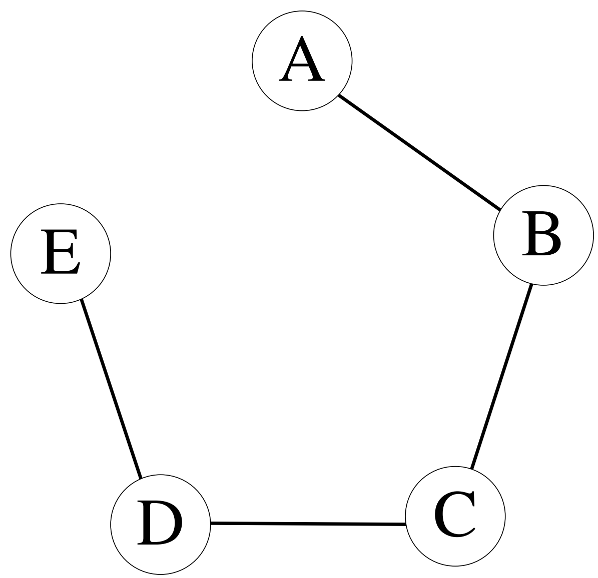


Depiction of graphs

Consider the graph with

- People: Avi, Benny, Cindy, Danny, Ella
- Pairs of people: $(A, B), (B, C), (C, D), (D, E), (E, A)$.

Good idea to draw it:

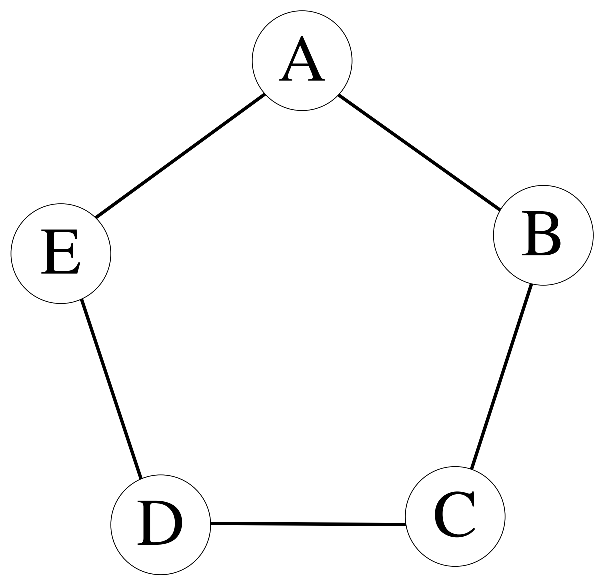


Depiction of graphs

Consider the graph with

- People: Avi, Benny, Cindy, Danny, Ella
- Pairs of people: $(A, B), (B, C), (C, D), (D, E), (E, A)$.

Good idea to draw it:

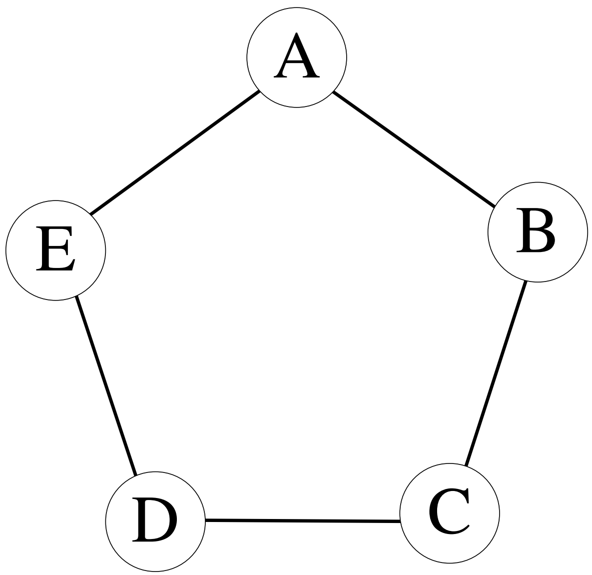


Different ways to depict graphs

The same cycle on 5 vertices can be drawn in many ways:

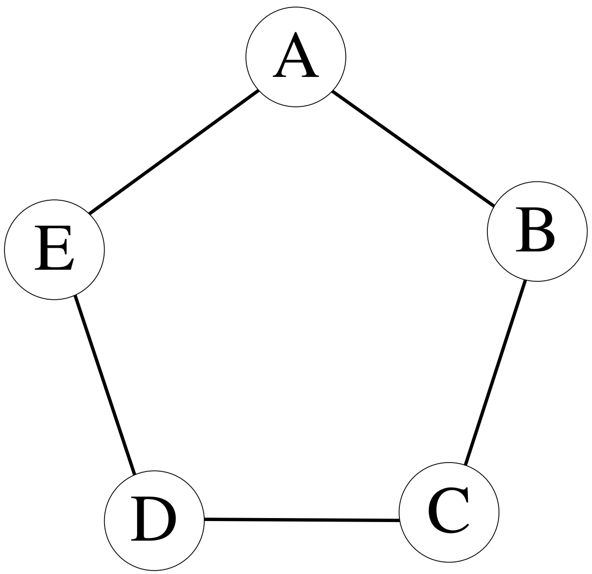
Different ways to depict graphs

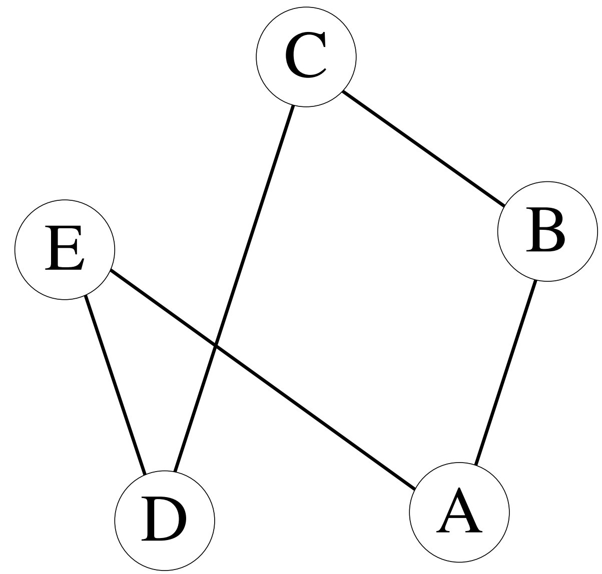
The same cycle on 5 vertices can be drawn in many ways:



Different ways to depict graphs

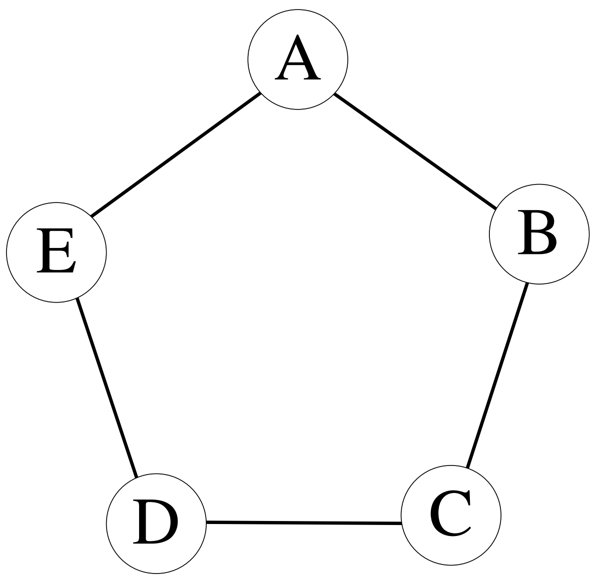
The same cycle on 5 vertices can be drawn in many ways:

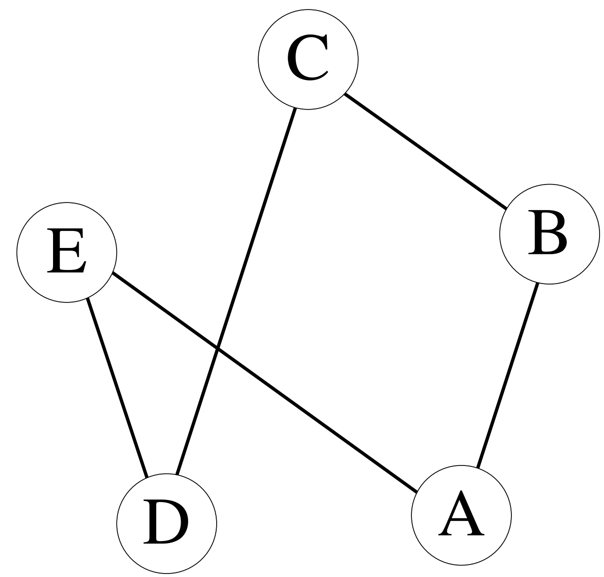




Different ways to depict graphs

The same cycle on 5 vertices can be drawn in many ways:

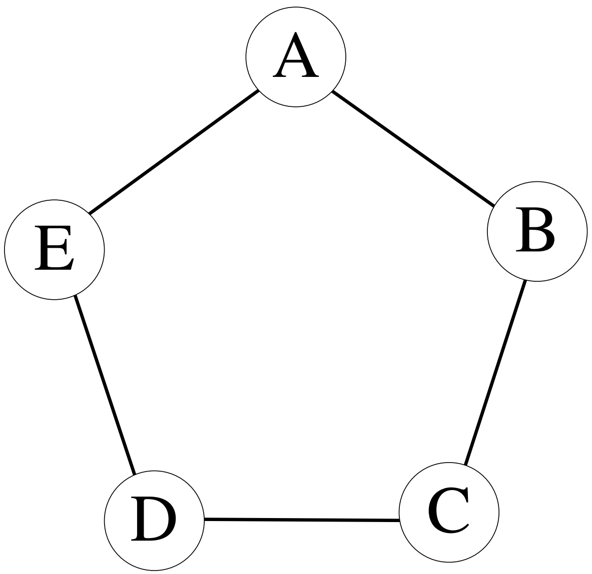


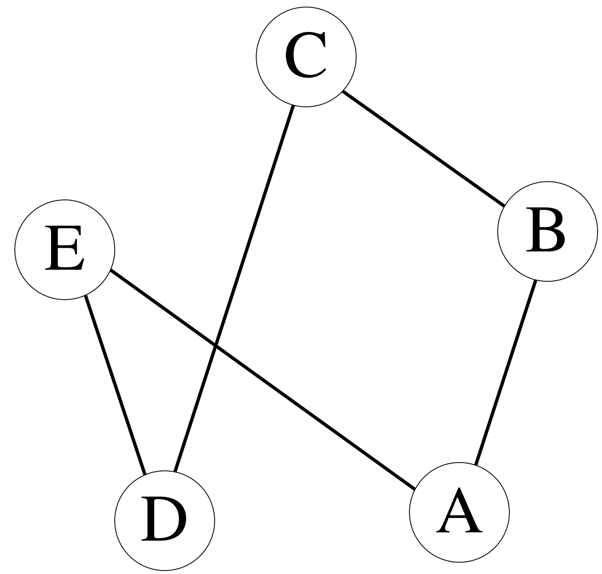


Question: Which drawing is “better”?

Different ways to depict graphs

The same cycle on 5 vertices can be drawn in many ways:





Question: Which drawing is "better"?

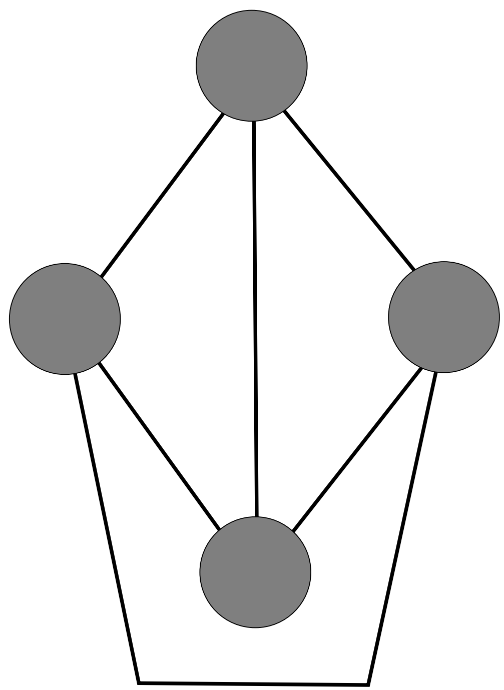
Answer: Prefer drawings without crossings of edges.

Planar Graphs

DEF: A graph is planar if it can be drawn so that no two edges cross.

Planar Graphs

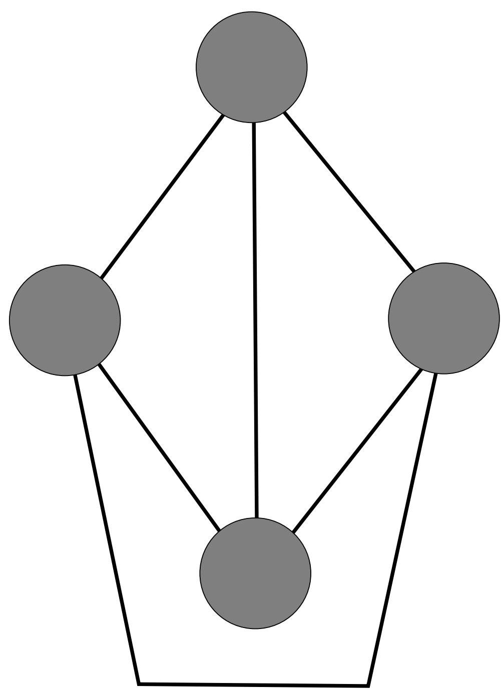
DEF: A graph is planar if it can be drawn so that no two edges cross.



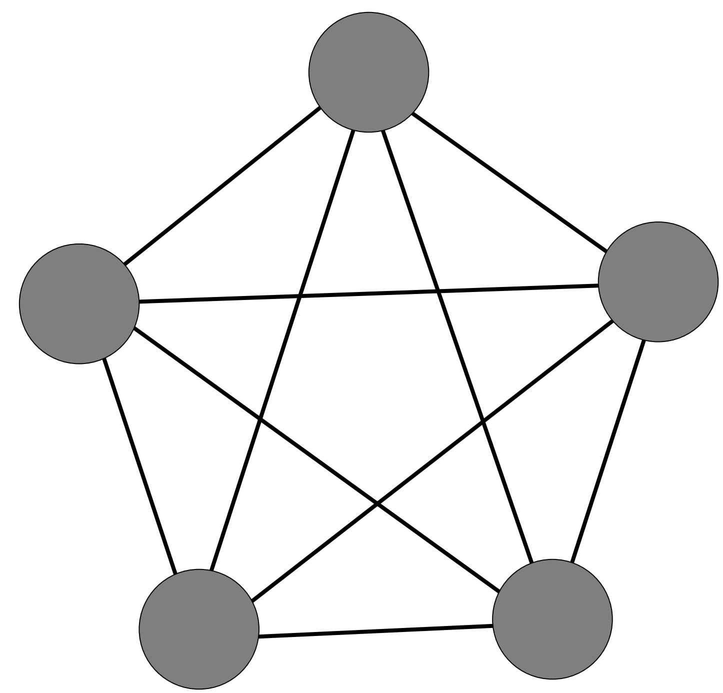
a planar graph

Planar Graphs

DEF: A graph is planar if it can be drawn so that no two edges cross.



a planar graph



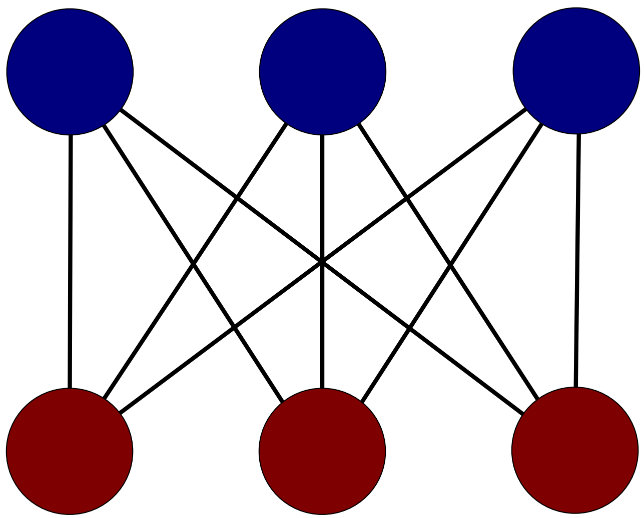
a non-planar graph

Characterization of planar graphs

THM: [Kuratowski 1930] A graph is planar if and only if it does not contain a “copy” of K_5 or $K_{3,3}$.

Characterization of planar graphs

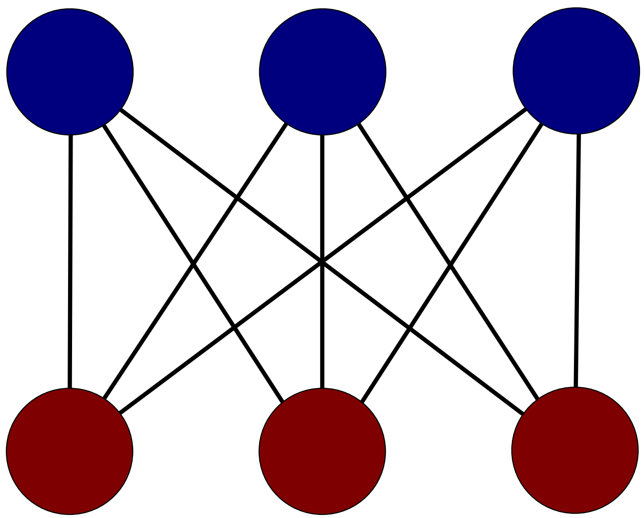
THM: [Kuratowski 1930] A graph is planar if and only if it does not contain a “copy” of K_5 or $K_{3,3}$.



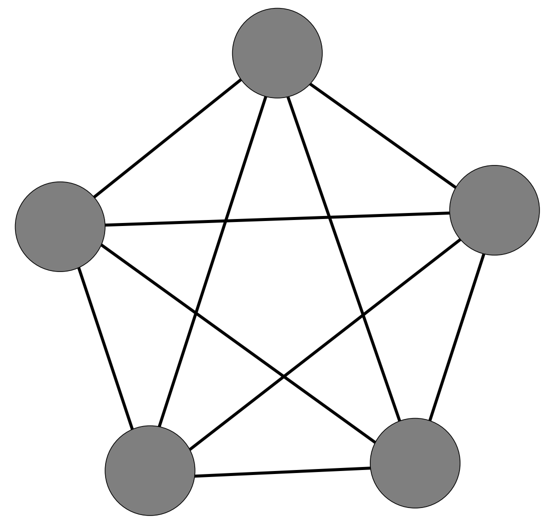
$K_{3,3}$

Characterization of planar graphs

THM: [Kuratowski 1930] A graph is planar if and only if it does not contain a "copy" of K_5 or $K_{3,3}$.



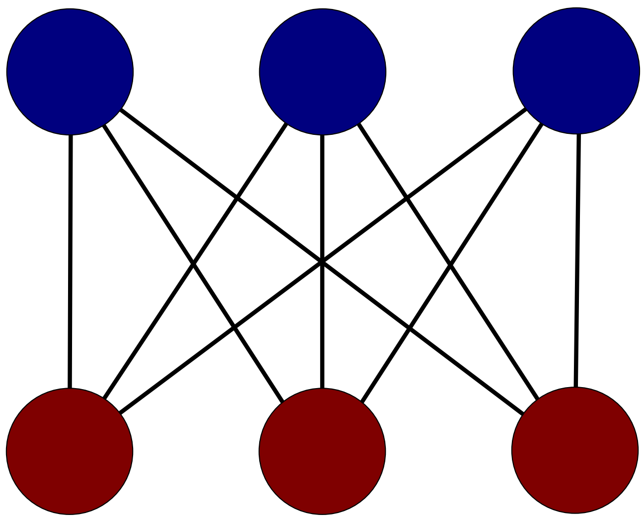
$K_{3,3}$



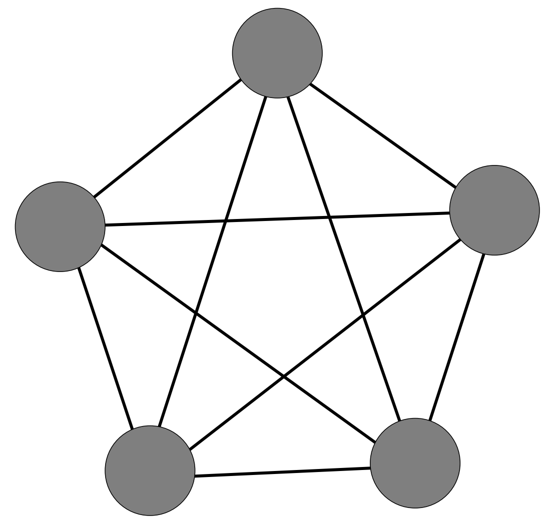
K_5

Characterization of planar graphs

THM: [Kuratowski 1930] A graph is planar if and only if it does not contain a “copy” of K_5 or $K_{3,3}$.



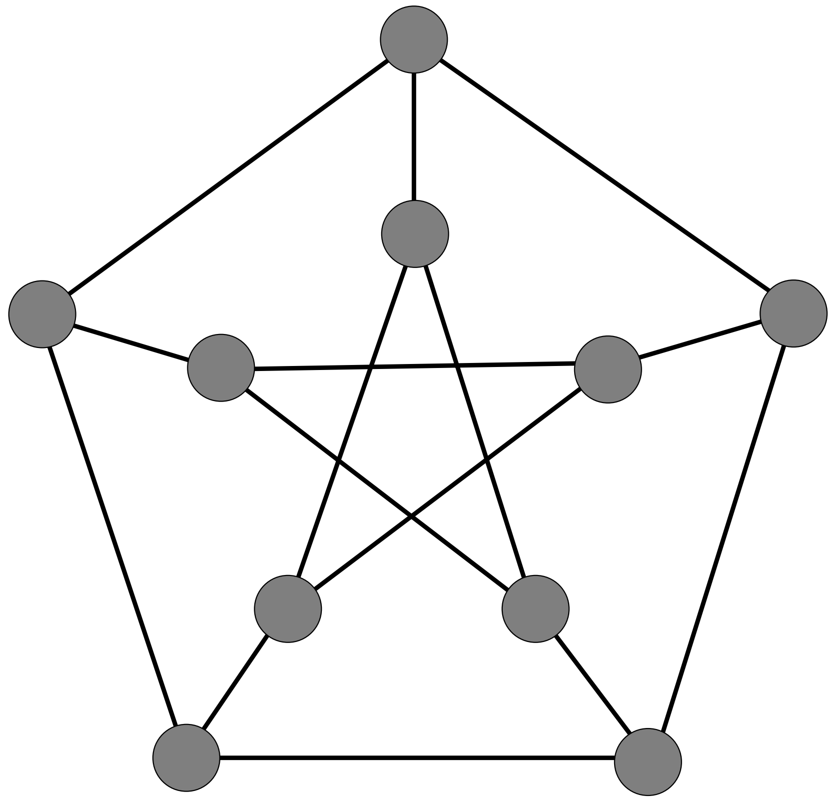
$K_{3,3}$



K_5

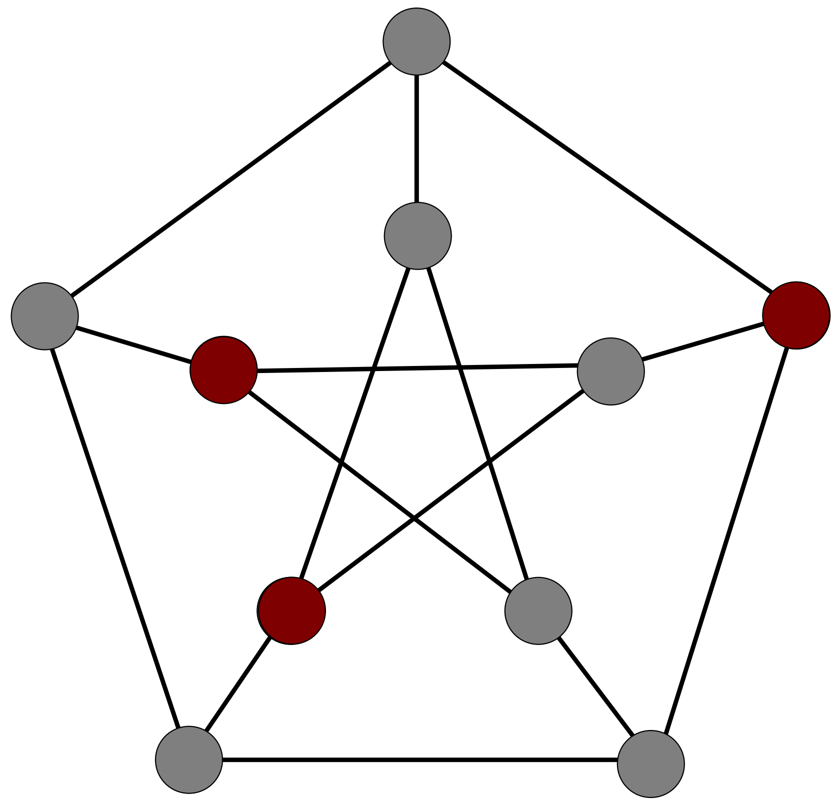
K_5 and $K_{3,3}$ are the “smallest” non-planar graphs and must “hide” in every non-planar graph. Often called forbidden graphs.

Is this graph planar?



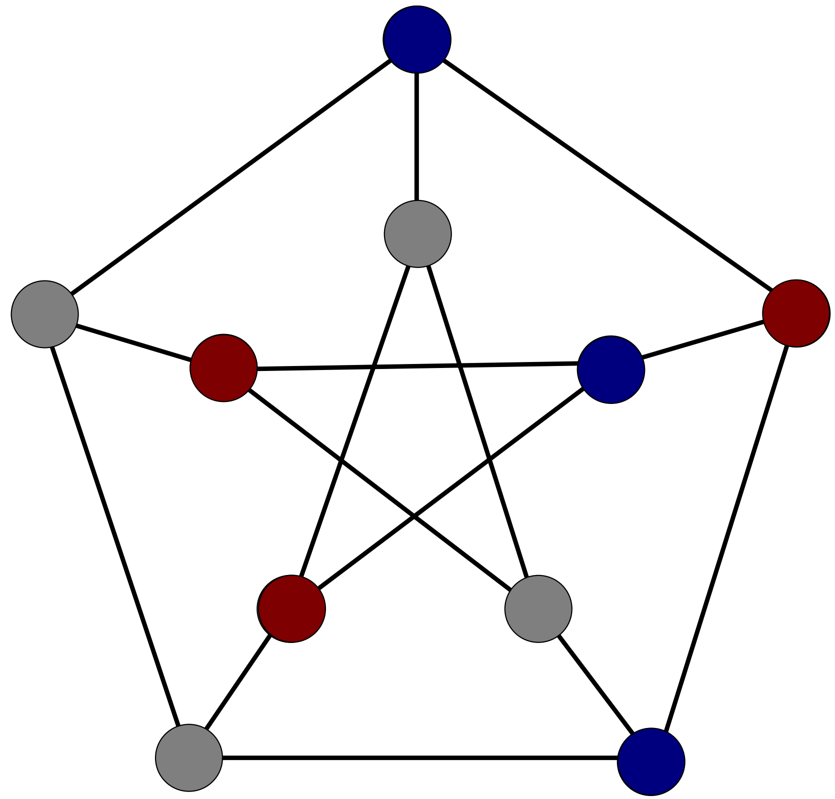
This is not a planar drawing - but does a planar drawing exist? Look for a copy of forbidden graph ($K_{3,3}$)...

Is this graph planar?



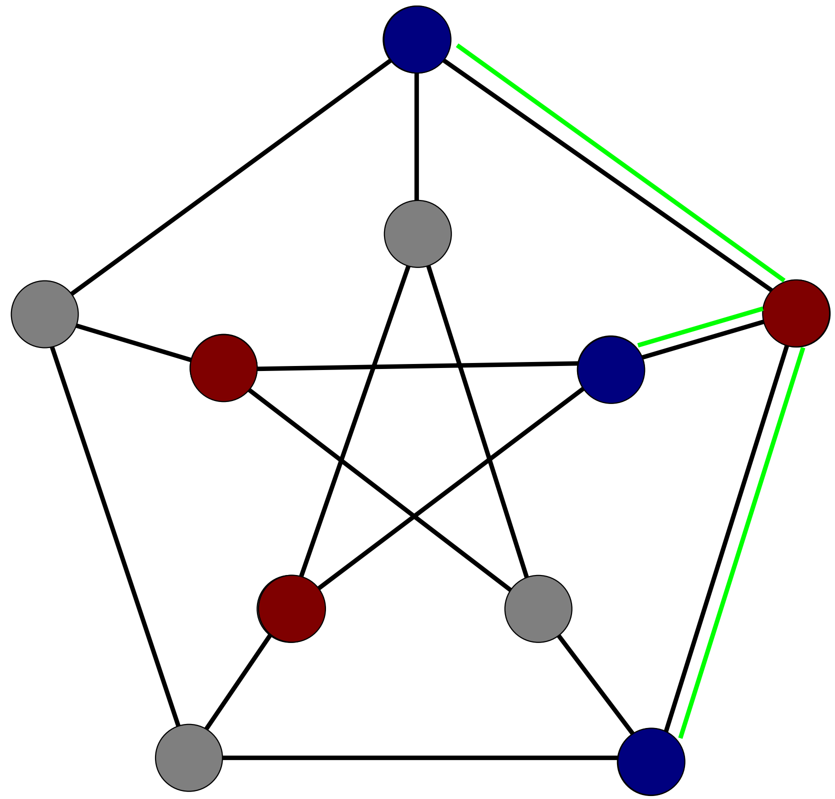
We mark the “red” nodes

Is this graph planar?



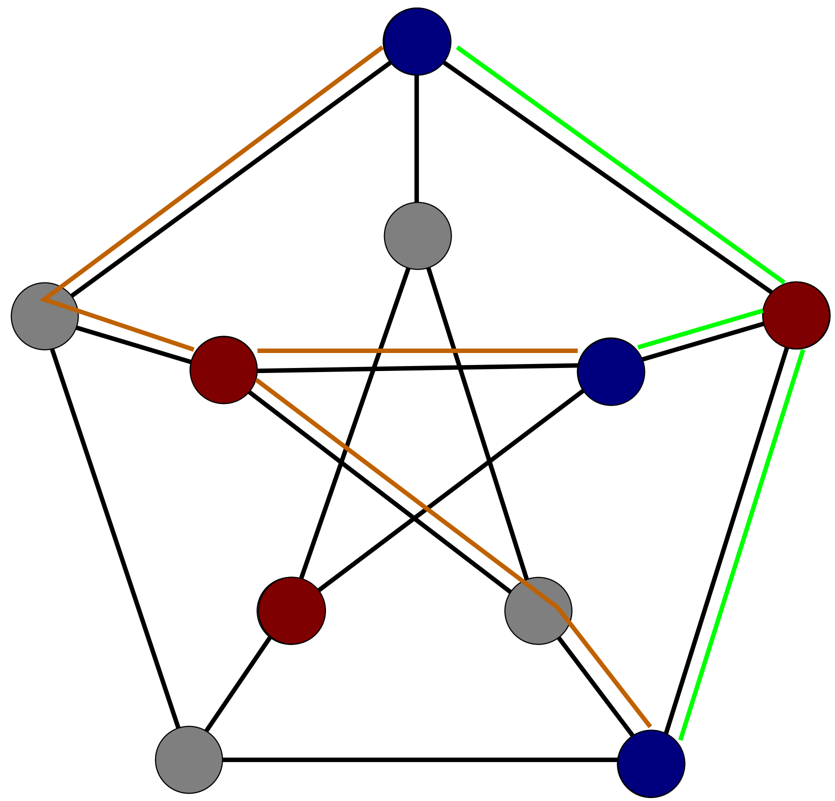
We mark the “blue” nodes

Is this graph planar?



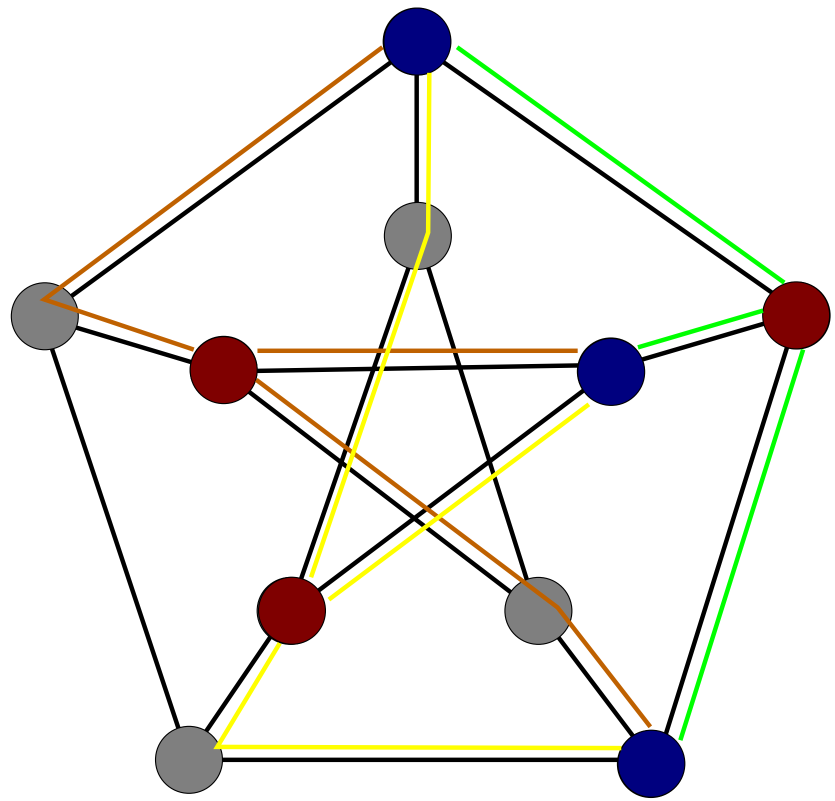
Mark paths from the 1st red node to the blue nodes

Is this graph planar?



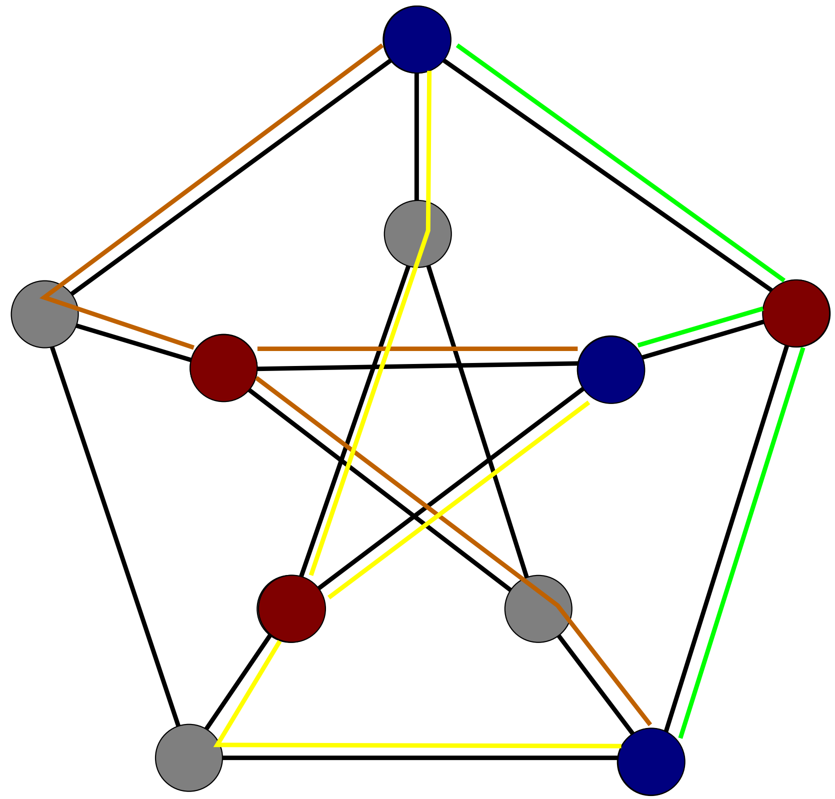
Mark paths from the 2nd red node to the blue nodes

Is this graph planar?



Mark paths from the 3rd red node to the blue nodes

Is this graph planar?



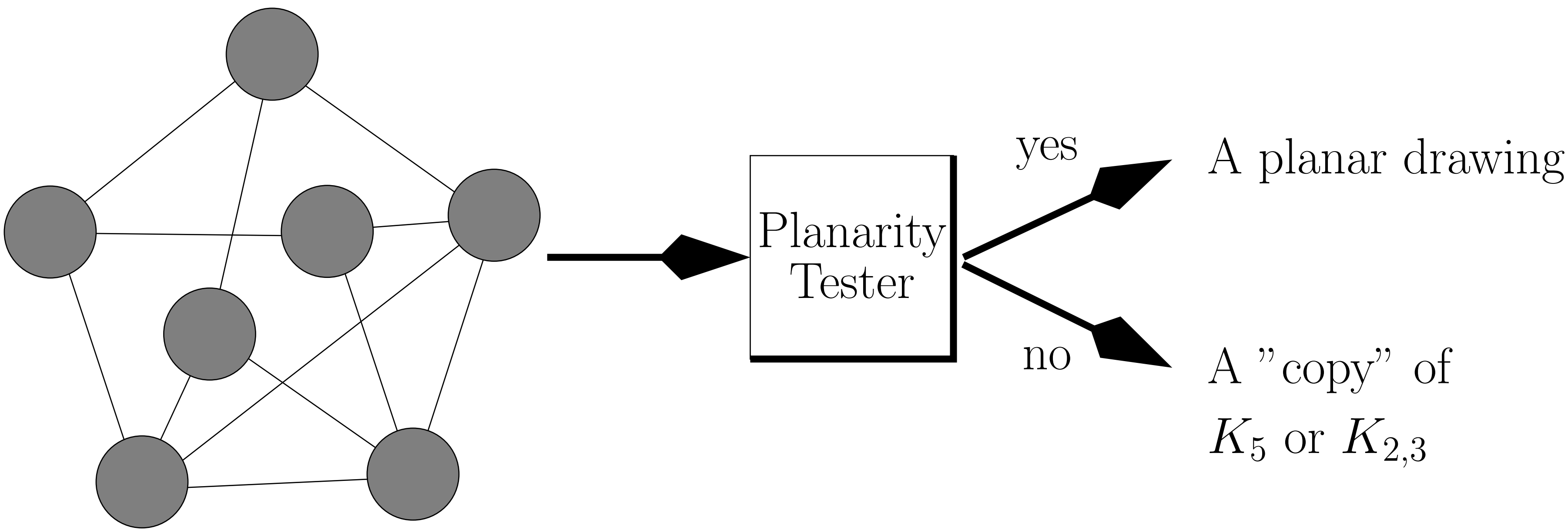
Mark paths from the 3rd red node to the blue nodes We found a “copy” of $K_{3,3} \Longrightarrow$ non-planar!

Planarity Testing

An algorithm for planarity testing:

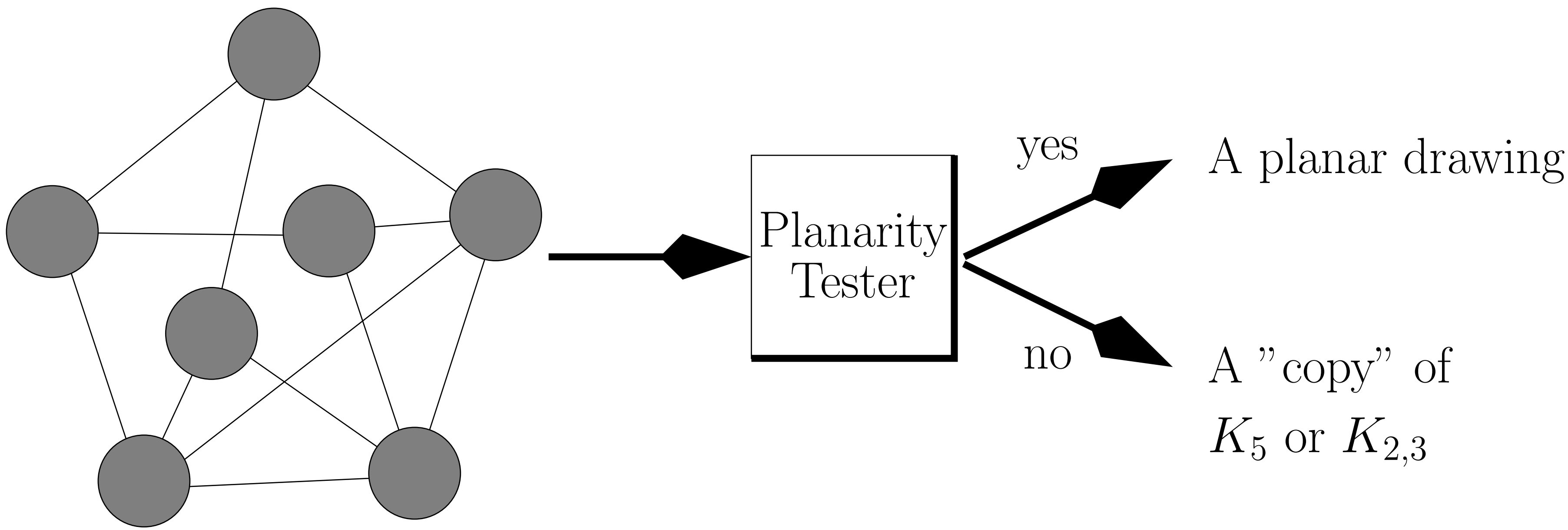
Planarity Testing

An algorithm for planarity testing:



Planarity Testing

An algorithm for planarity testing:



[Lempel, Even, & Cederbaum 1967]:

A polynomial time algorithm for planarity testing. Linear time realizations by [Even & Tarjan 76, Booth & Lueker 76].

Why investigate graph drawing?

- Aesthetic topic.

Why investigate graph drawing?

- Aesthetic topic.
- Major theme in combinatorics and graph theory (draw a graph with fewest possible edge crossings).

Why investigate graph drawing?

- Aesthetic topic.
- Major theme in combinatorics and graph theory (draw a graph with fewest possible edge crossings).
- Applications in printed circuits and VLSI design.

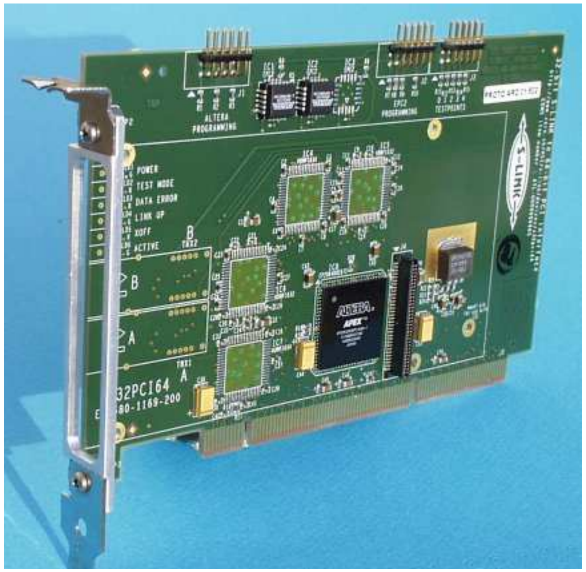
Why investigate graph drawing?

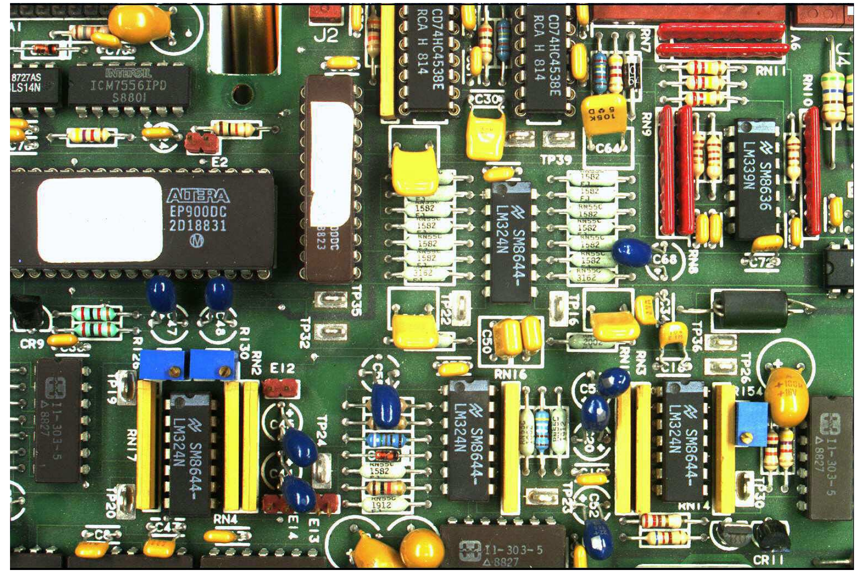
- Aesthetic topic.
- Major theme in combinatorics and graph theory (draw a graph with fewest possible edge crossings).
- Applications in printed circuits and VLSI design.
- Price of production of VLSI chip is proportional to chip area.

Why investigate graph drawing?

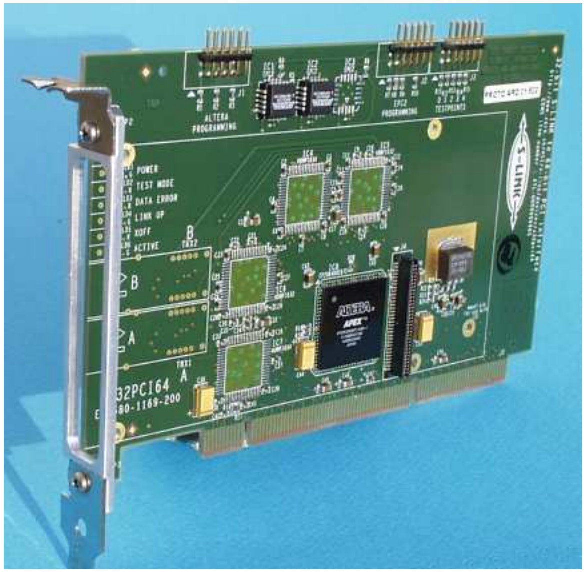
- Aesthetic topic.
- Major theme in combinatorics and graph theory (draw a graph with fewest possible edge crossings).
- Applications in printed circuits and VLSI design.
- Price of production of VLSI chip is proportional to chip area.
- $\longrightarrow$ Want to find drawings with smallest possible area...

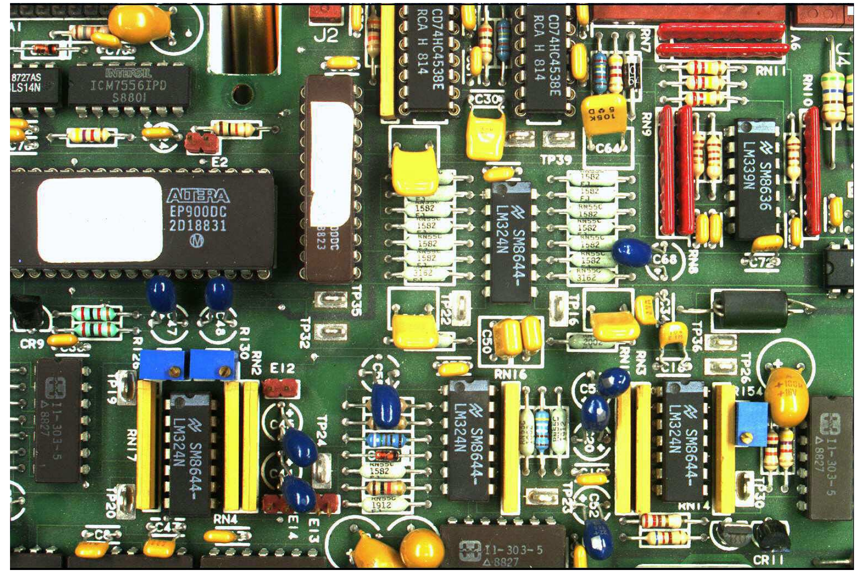
Printed Circuit Boards

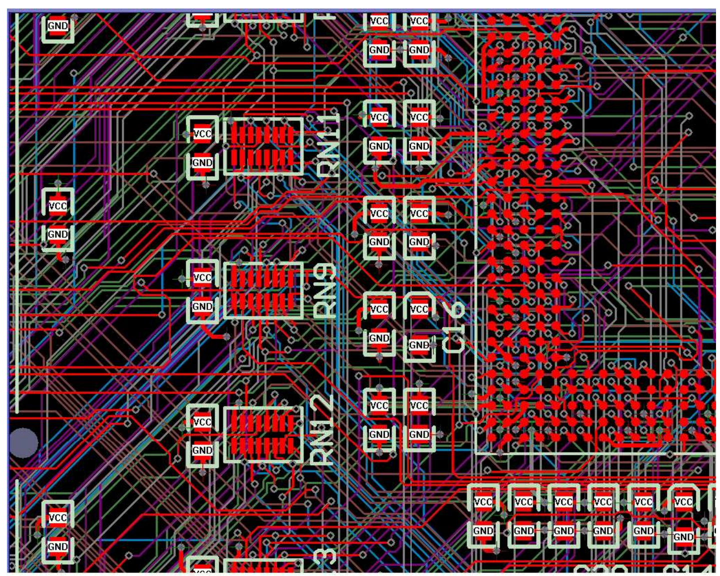




Printed Circuit Boards





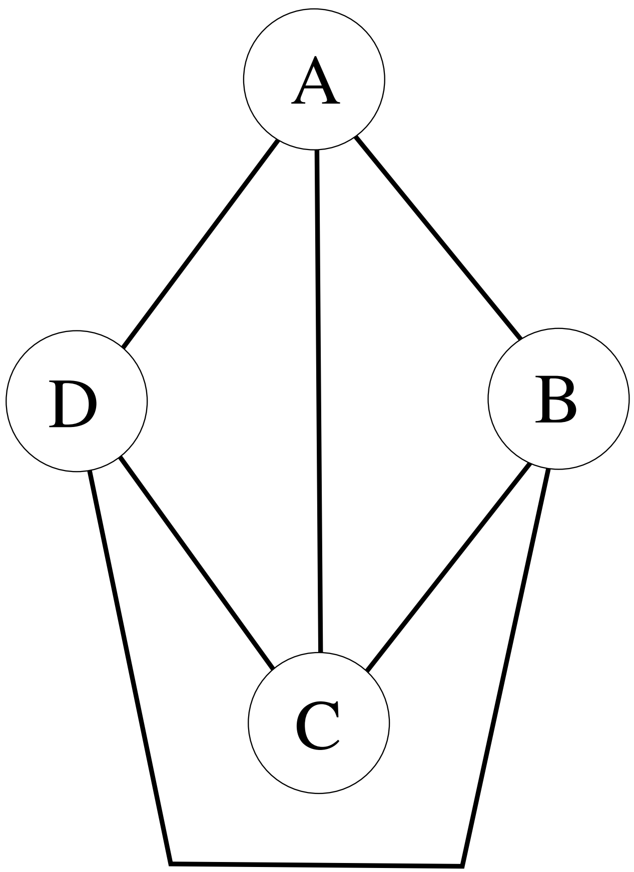


Layout of graph

Topic invented in PhD thesis of Yossi Shiloach [1976]: Models printed circuits and VLSI chips [c.f. Thompson 1979].

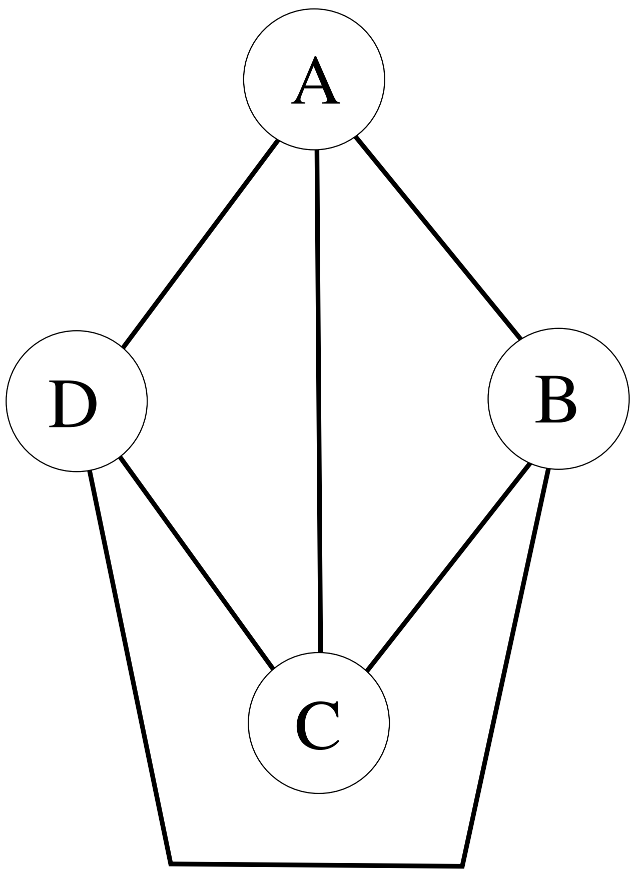
Layout of graph

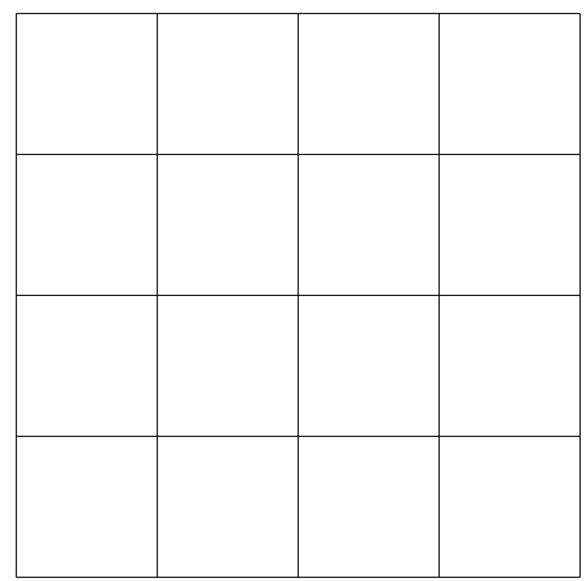
Topic invented in PhD thesis of Yossi Shiloach [1976]: Models printed circuits and VLSI chips [c.f. Thompson 1979].



Layout of graph

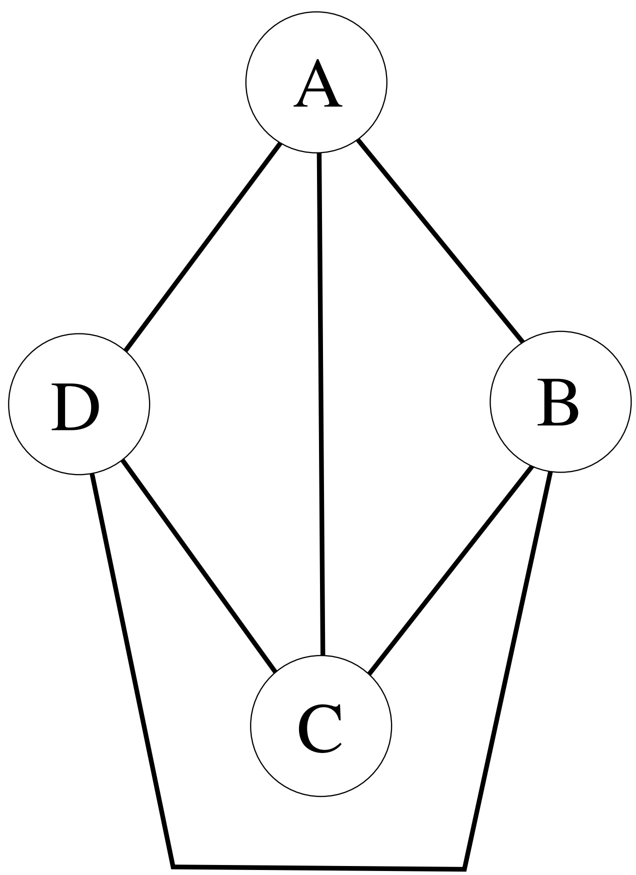
Topic invented in PhD thesis of Yossi Shiloach [1976]: Models printed circuits and VLSI chips [c.f. Thompson 1979].

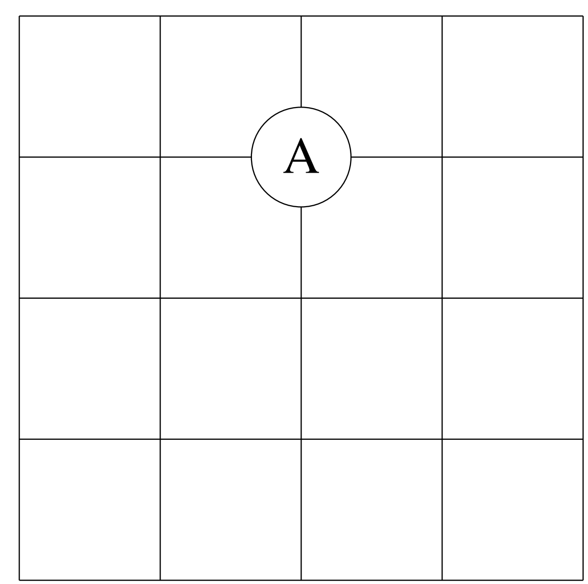




Layout of graph

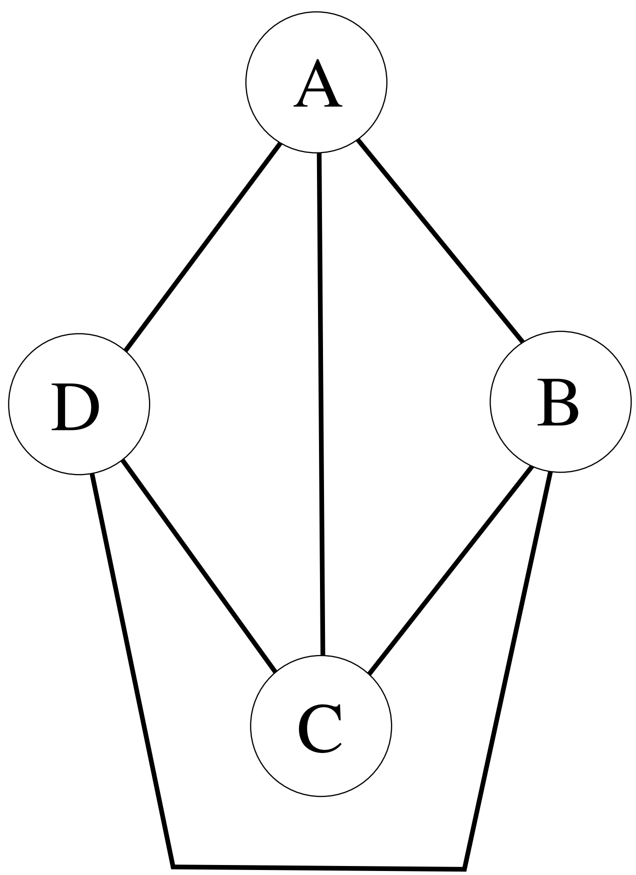
Topic invented in PhD thesis of Yossi Shiloach [1976]:
Models printed circuits and VLSI chips [c.f. Thompson 1979].

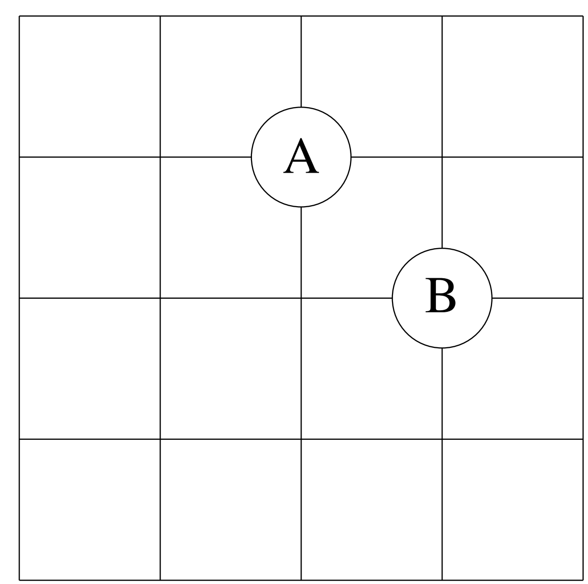




Layout of graph

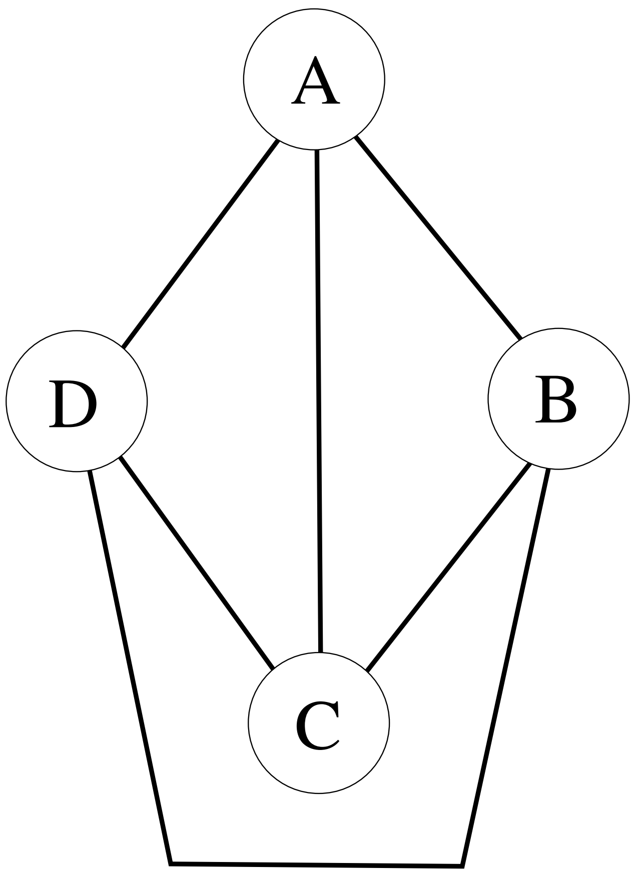
Topic invented in PhD thesis of Yossi Shiloach [1976]: Models printed circuits and VLSI chips [c.f. Thompson 1979].

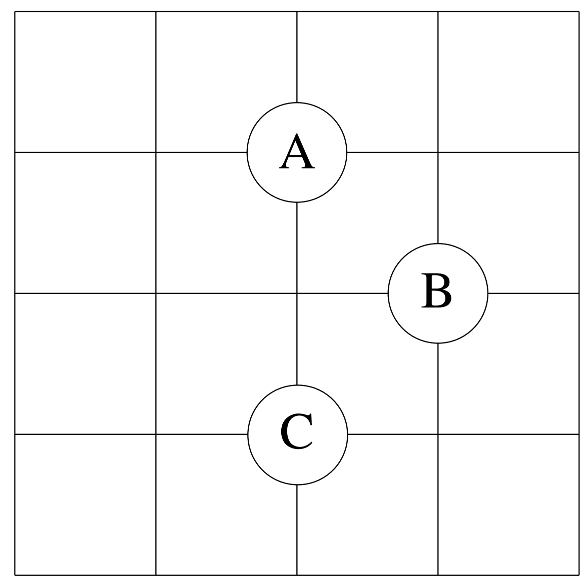




Layout of graph

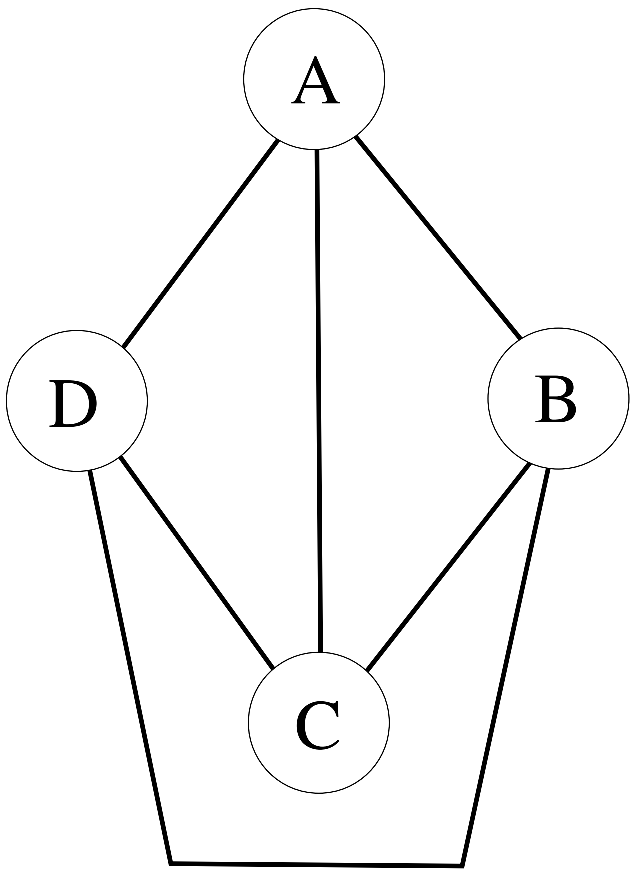
Topic invented in PhD thesis of Yossi Shiloach [1976]: Models printed circuits and VLSI chips [c.f. Thompson 1979].

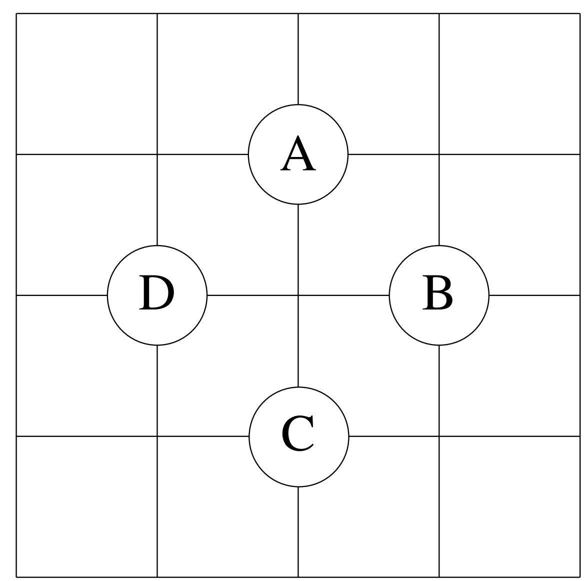




Layout of graph

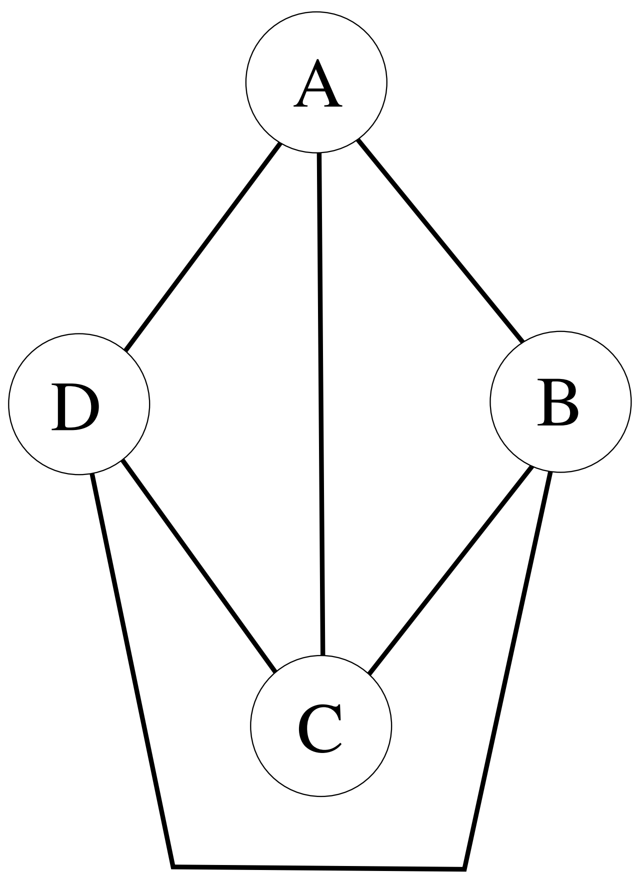
Topic invented in PhD thesis of Yossi Shiloach [1976]: Models printed circuits and VLSI chips [c.f. Thompson 1979].

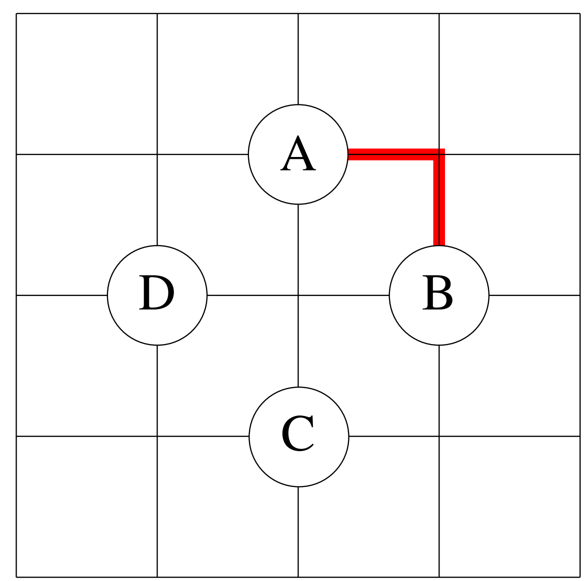




Layout of graph

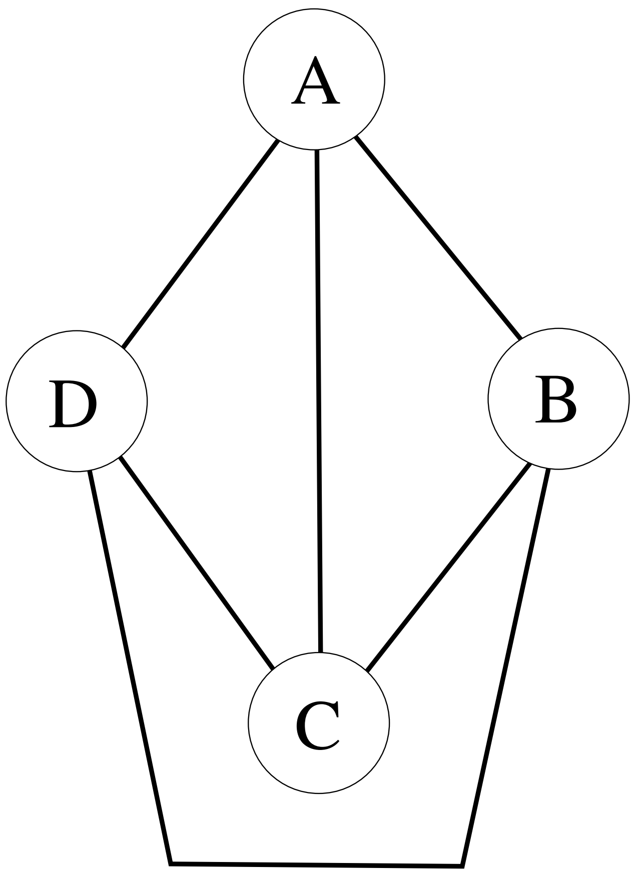
Topic invented in PhD thesis of Yossi Shiloach [1976]: Models printed circuits and VLSI chips [c.f. Thompson 1979].

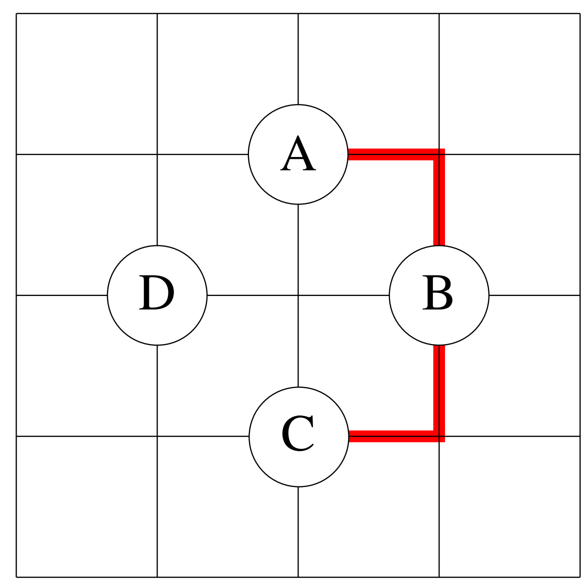




Layout of graph

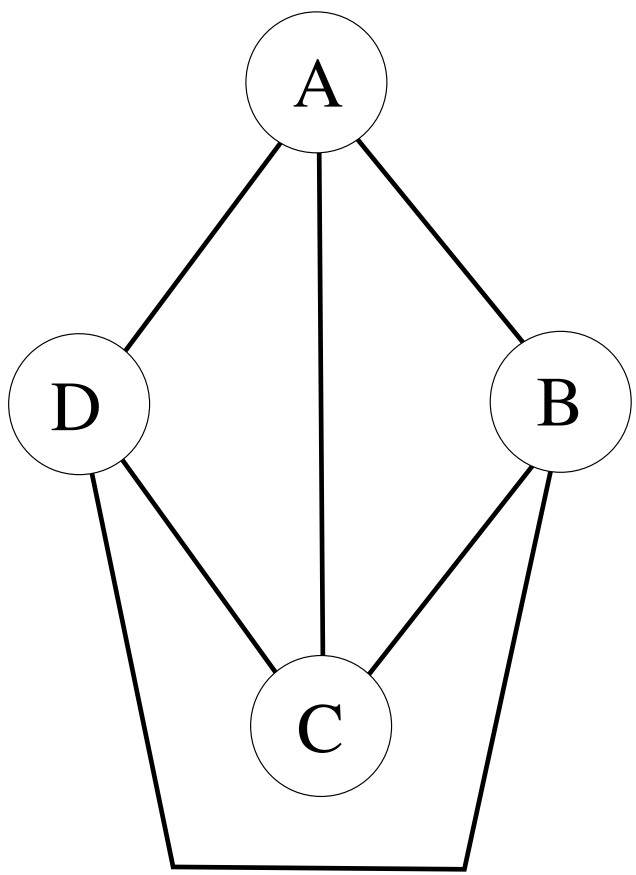
Topic invented in PhD thesis of Yossi Shiloach [1976]: Models printed circuits and VLSI chips [c.f. Thompson 1979].

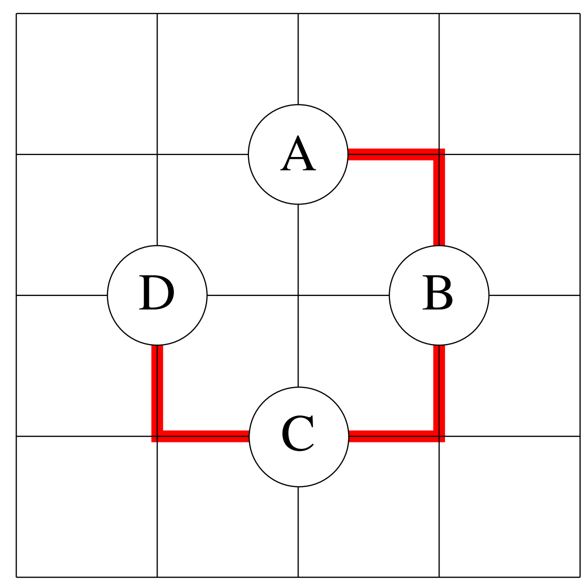




Layout of graph

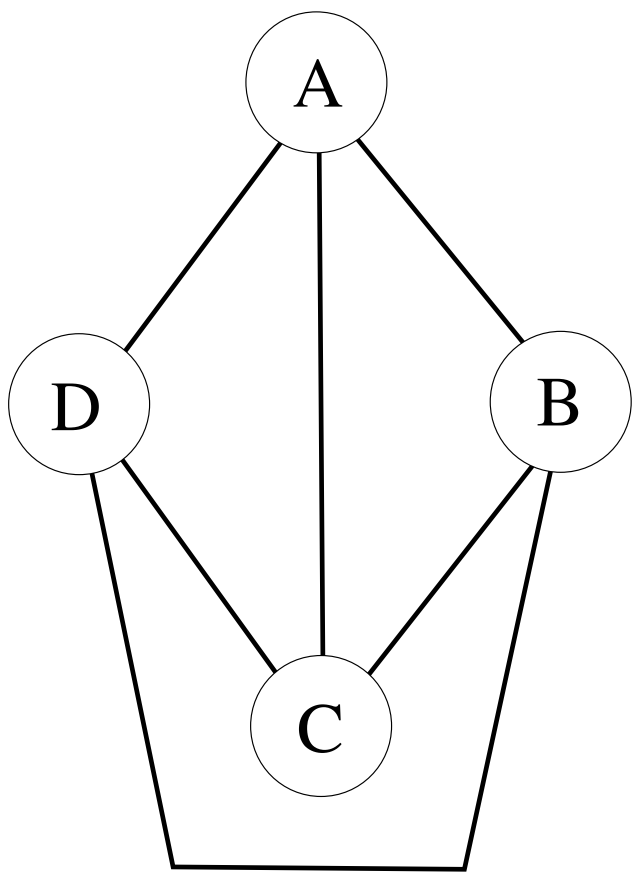
Topic invented in PhD thesis of Yossi Shiloach [1976]: Models printed circuits and VLSI chips [c.f. Thompson 1979].

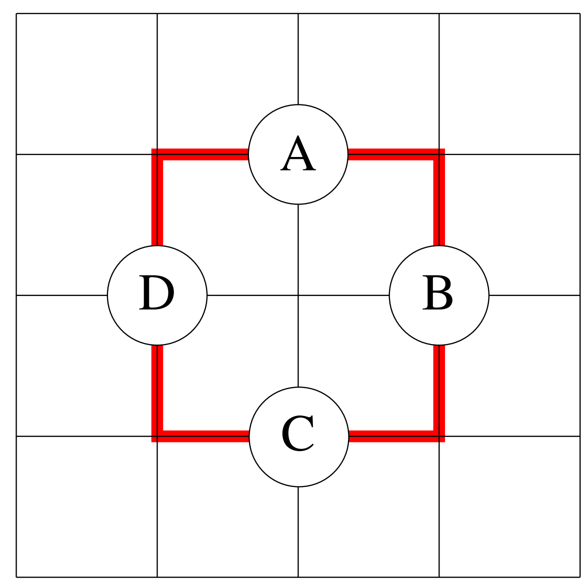




Layout of graph

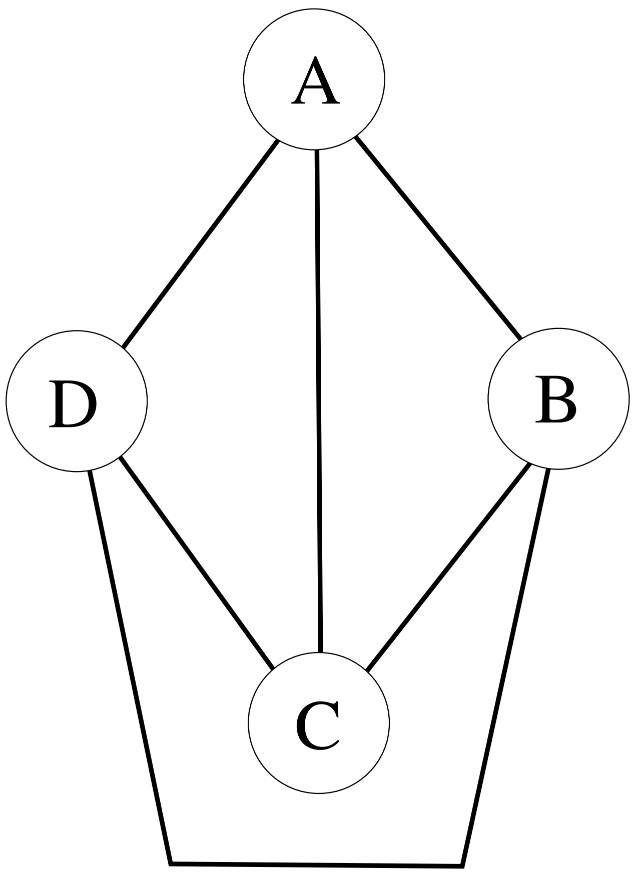
Topic invented in PhD thesis of Yossi Shiloach [1976]: Models printed circuits and VLSI chips [c.f. Thompson 1979].

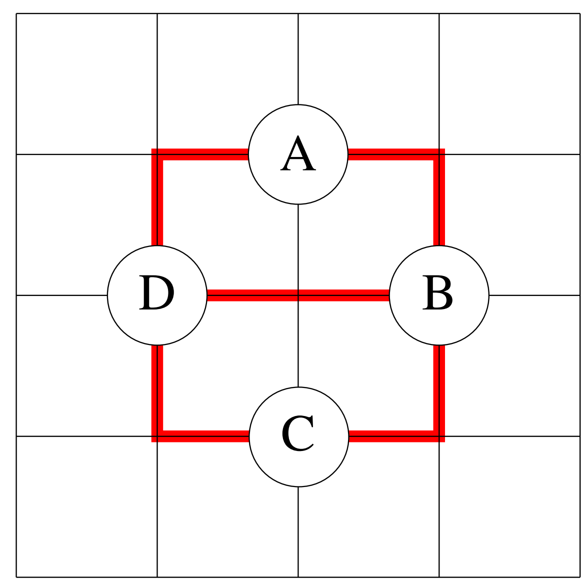




Layout of graph

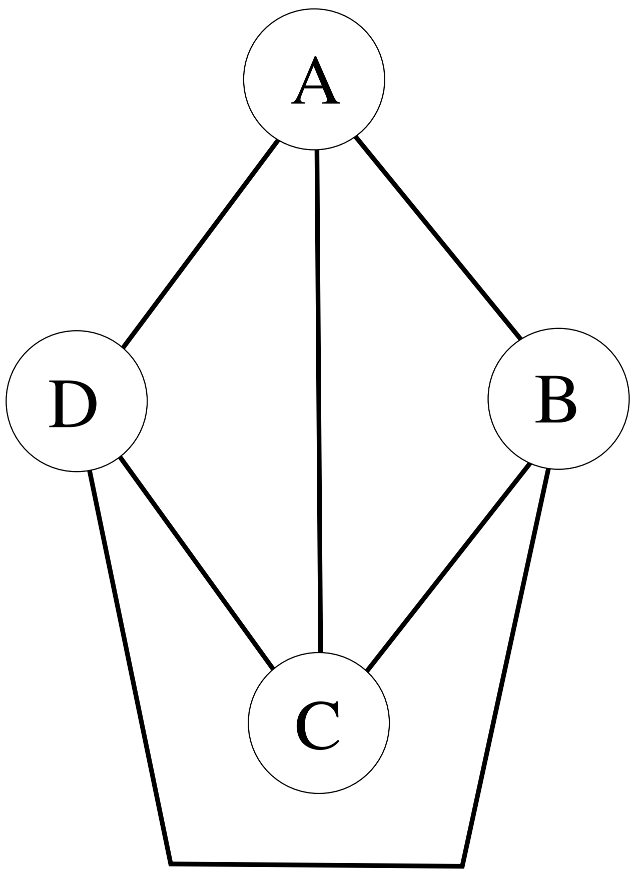
Topic invented in PhD thesis of Yossi Shiloach [1976]: Models printed circuits and VLSI chips [c.f. Thompson 1979].

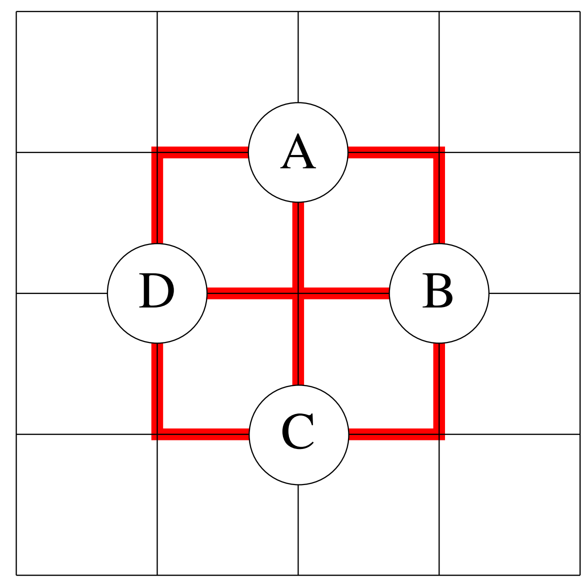




Layout of graph

Topic invented in PhD thesis of Yossi Shiloach [1976]: Models printed circuits and VLSI chips [c.f. Thompson 1979].





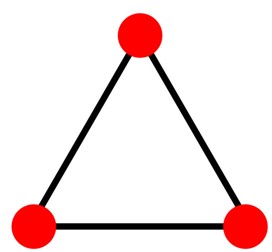
Planarity & Layout

Question: Do planar drawings of planar graphs lead to small area layout?
Example: [Shiloach] Nested triangles...

Planarity & Layout

Question: Do planar drawings of planar graphs lead to small area layout?

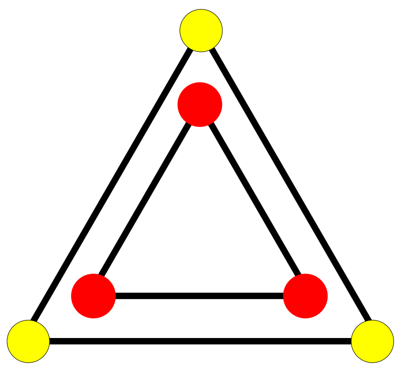
Example: [Shiloach] Nested triangles...



Planarity & Layout

Question: Do planar drawings of planar graphs lead to small area layout?

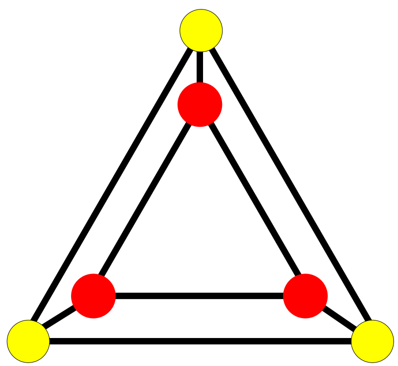
Example: [Shiloach] Nested triangles...



Planarity & Layout

Question: Do planar drawings of planar graphs lead to small area layout?

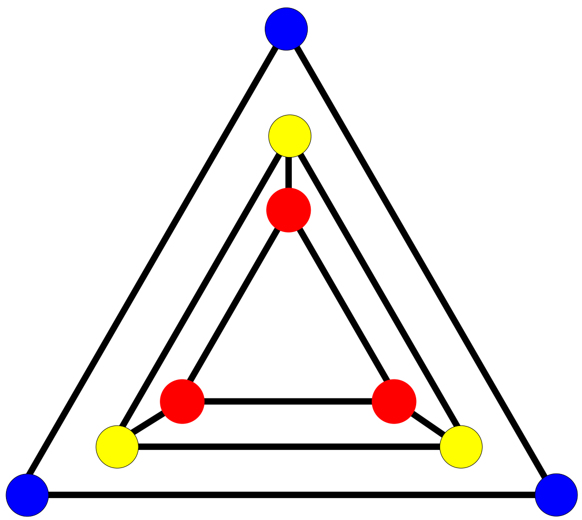
Example: [Shiloach] Nested triangles...



Planarity & Layout

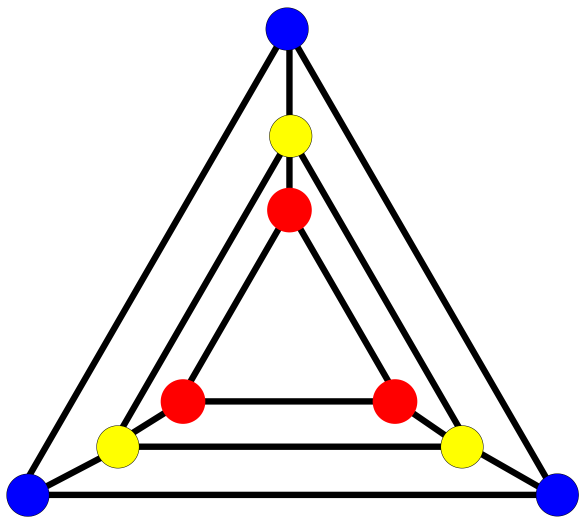
Question: Do planar drawings of planar graphs lead to small area layout?

Example: [Shiloach] Nested triangles...



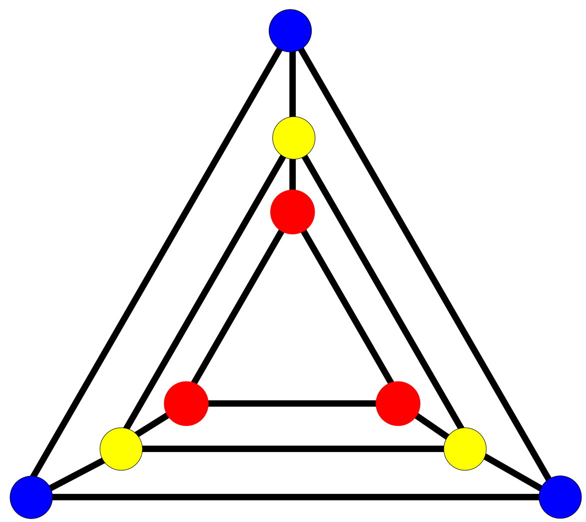
Planarity & Layout

Question: Do planar drawings of planar graphs lead to small area layout?
Example: [Shiloach] Nested triangles...



Planarity & Layout

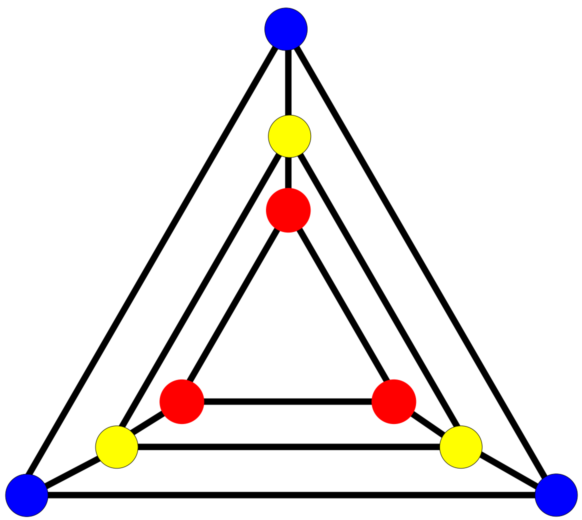
Question: Do planar drawings of planar graphs lead to small area layout?
Example: [Shiloach] Nested triangles...



- planar graph & drawing

Planarity & Layout

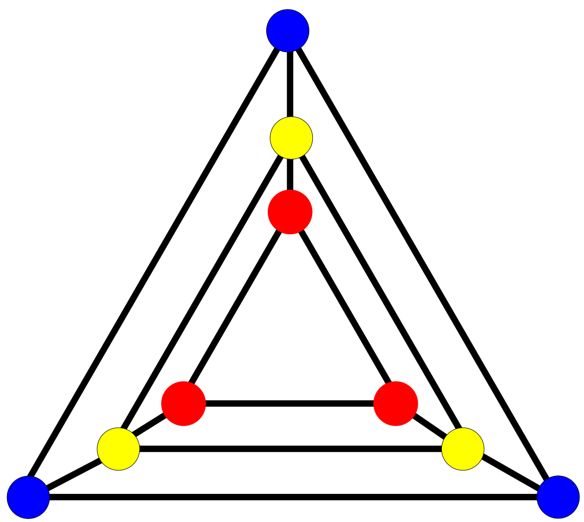
Question: Do planar drawings of planar graphs lead to small area layout?
Example: [Shiloach] Nested triangles...



- planar graph & drawing
- every additional triangle adds linear area to drawing

Planarity & Layout

Question: Do planar drawings of planar graphs lead to small area layout?
Example: [Shiloach] Nested triangles...

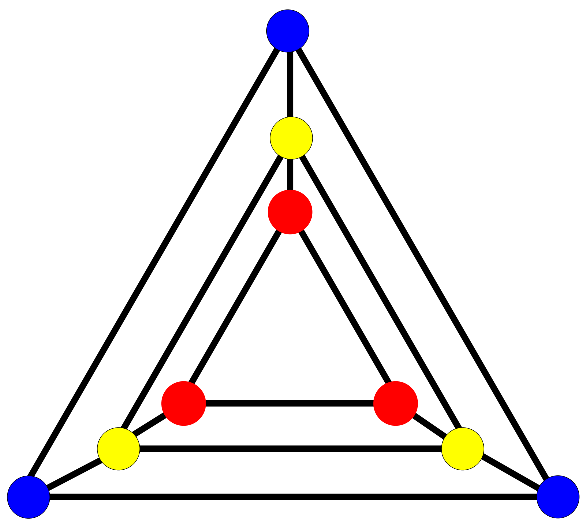


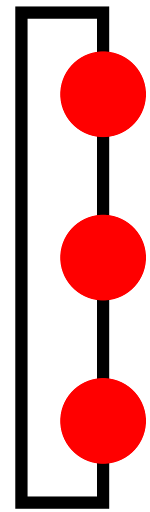
- planar graph & drawing
- every additional triangle adds linear area to drawing
- $\Longrightarrow$ area $= n^2$

Planarity & Layout

Question: Do planar drawings of planar graphs lead to small area layout?

Example: [Shiloach] Nested triangles...

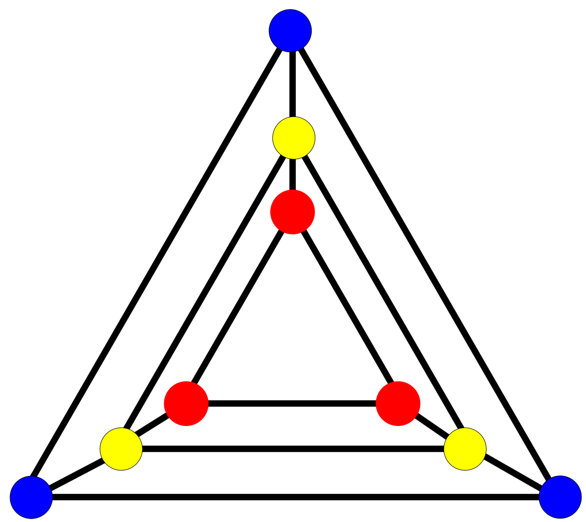


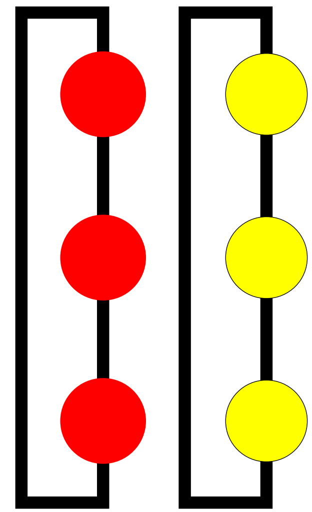


- planar graph & drawing
- every additional triangle adds linear area to drawing
- $\Longrightarrow$ area $= n^2$

Planarity & Layout

Question: Do planar drawings of planar graphs lead to small area layout?
Example: [Shiloach] Nested triangles...

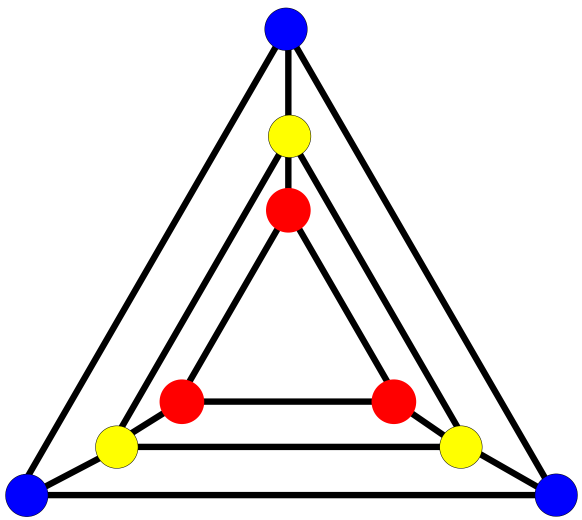


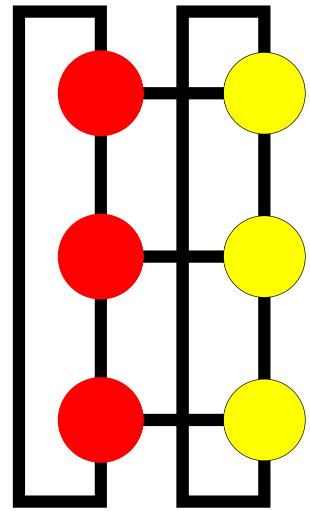


- planar graph & drawing
- every additional triangle adds linear area to drawing
- $\Longrightarrow$ area $= n^2$

Planarity & Layout

Question: Do planar drawings of planar graphs lead to small area layout?
Example: [Shiloach] Nested triangles...



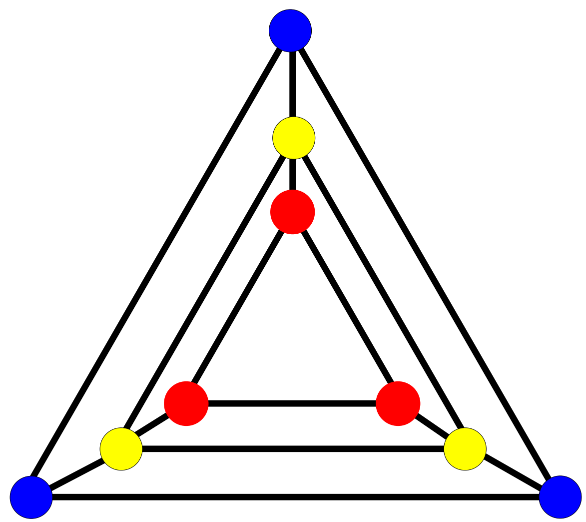


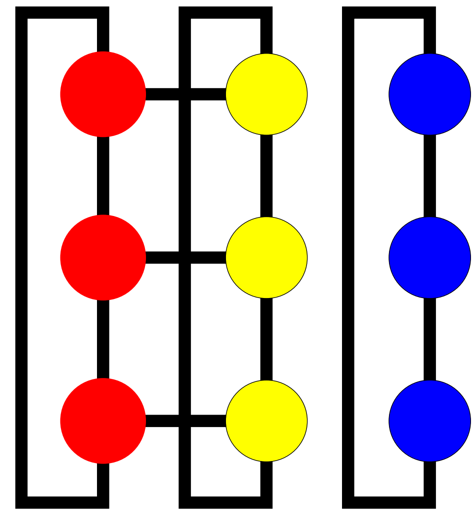
- planar graph & drawing
- every additional triangle adds linear area to drawing
- $\Longrightarrow$ area $= n^2$

Planarity & Layout

Question: Do planar drawings of planar graphs lead to small area layout?

Example: [Shiloach] Nested triangles...



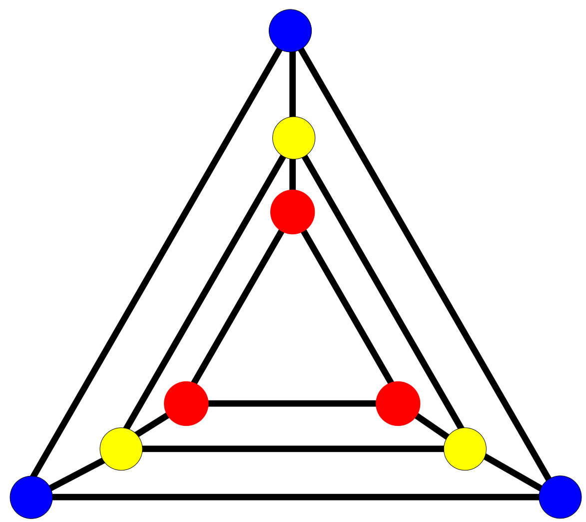


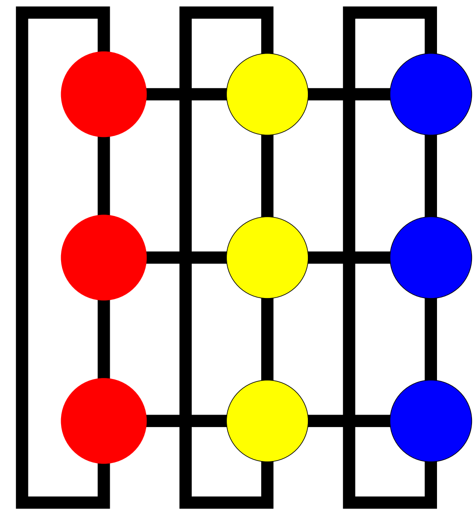
- planar graph & drawing
- every additional triangle adds linear area to drawing
- $\Longrightarrow$ area $= n^2$

Planarity & Layout

Question: Do planar drawings of planar graphs lead to small area layout?

Example: [Shiloach] Nested triangles...

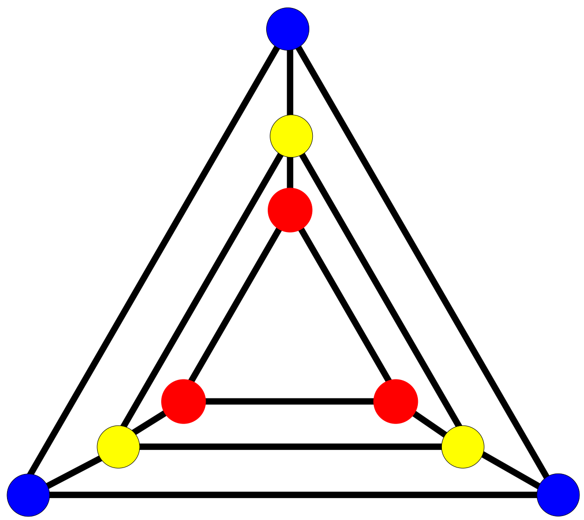


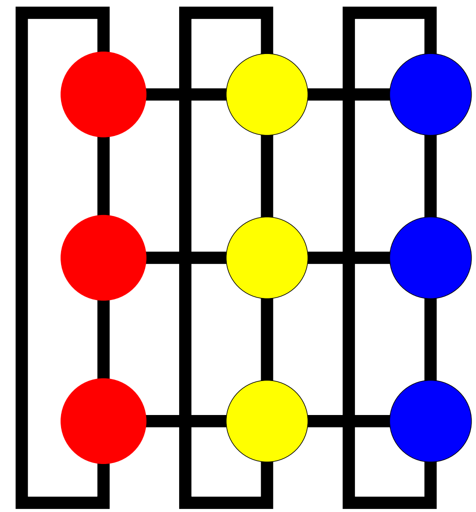


- planar graph & drawing
- every additional triangle adds linear area to drawing
- $\Longrightarrow$ area $= n^2$

Planarity & Layout

Question: Do planar drawings of planar graphs lead to small area layout?
Example: [Shiloach] Nested triangles...



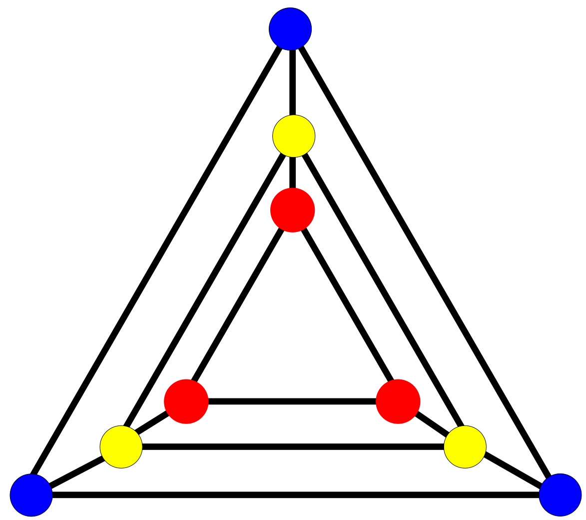


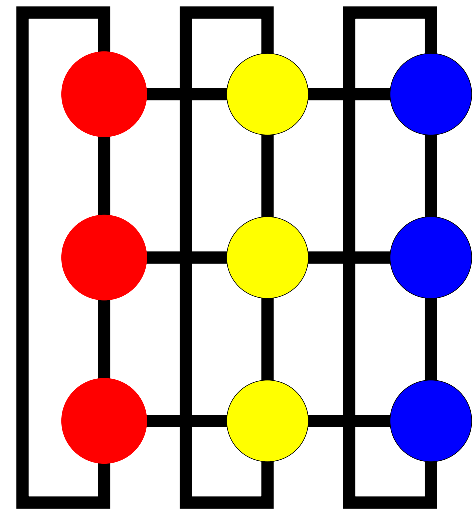
- planar graph & drawing
- every additional triangle adds linear area to drawing
- $\Longrightarrow$ area $= n^2$

- non-planar drawing

Planarity & Layout

Question: Do planar drawings of planar graphs lead to small area layout?
Example: [Shiloach] Nested triangles...



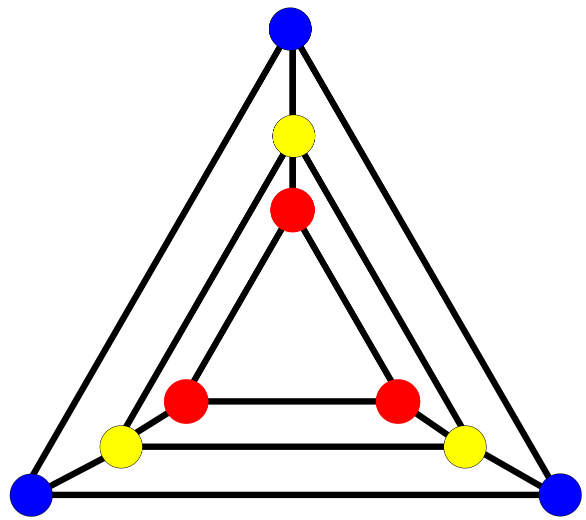


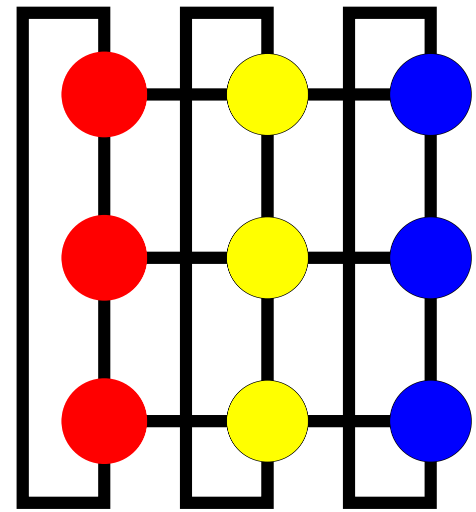
- planar graph & drawing
- every additional triangle adds linear area to drawing
- $\Longrightarrow$ area $= n^2$

- non-planar drawing
- every additional triangle adds constant area to drawing

Planarity & Layout

Question: Do planar drawings of planar graphs lead to small area layout?
Example: [Shiloach] Nested triangles...

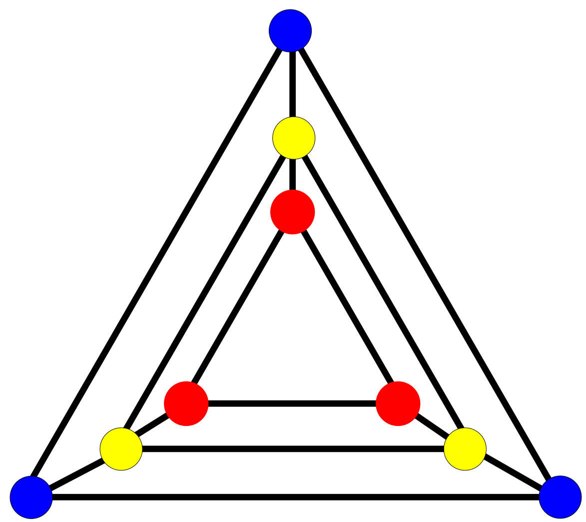


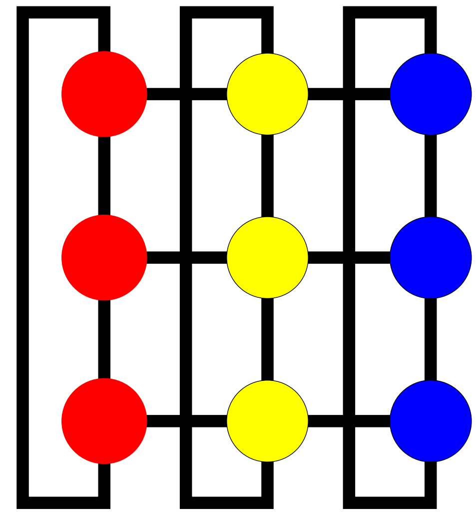


- planar graph & drawing
- every additional triangle adds linear area to drawing
- $\Longrightarrow$ area $= n^2$

- non-planar drawing
- every additional triangle adds constant area to drawing
- $\Longrightarrow$ area $= n$

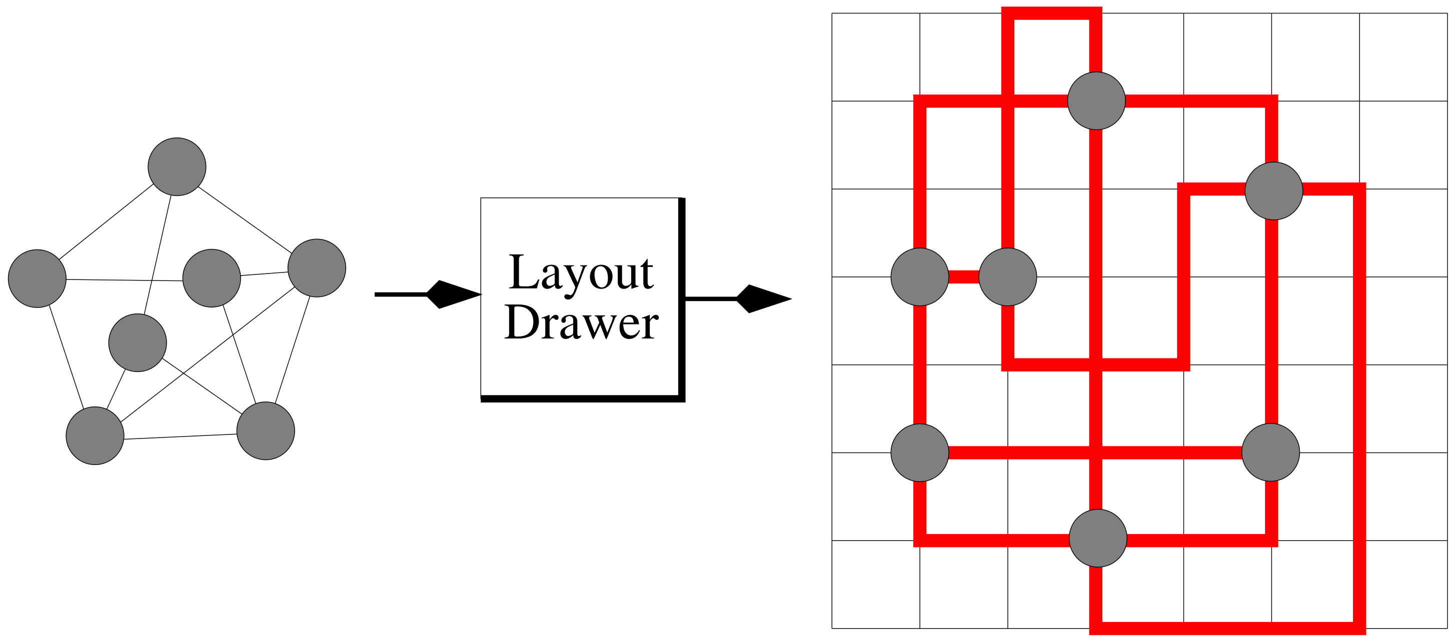
Planarity & Layouts



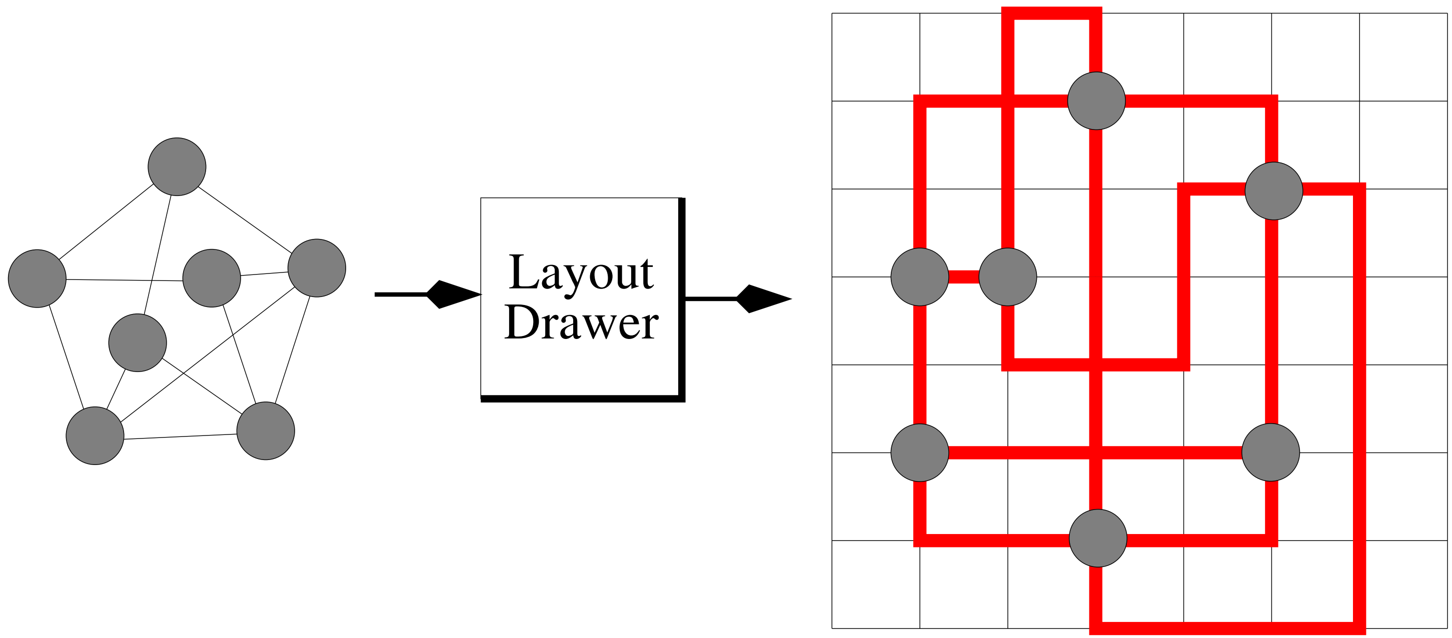


Conclusion: Using planar drawings is not always a good idea... Layout problem is much harder!

Dealing with the hardness of finding small area layouts

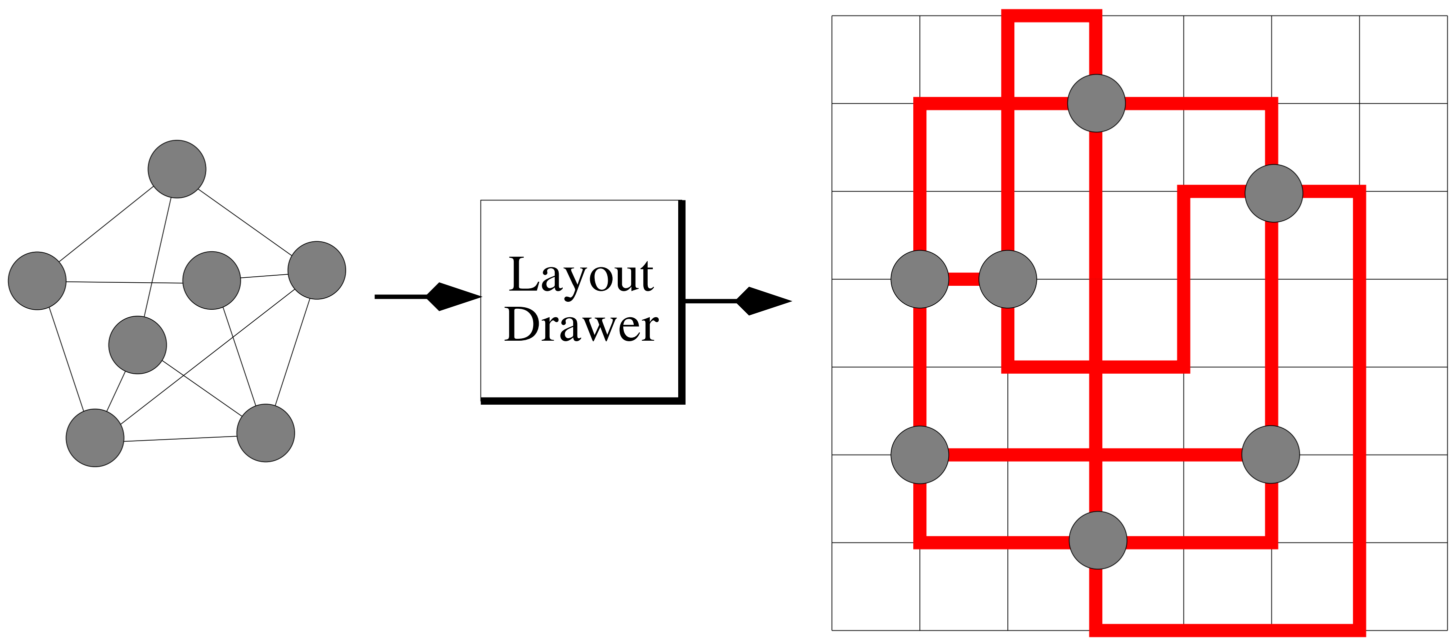


Dealing with the hardness of finding small area layouts



Bend the rules!

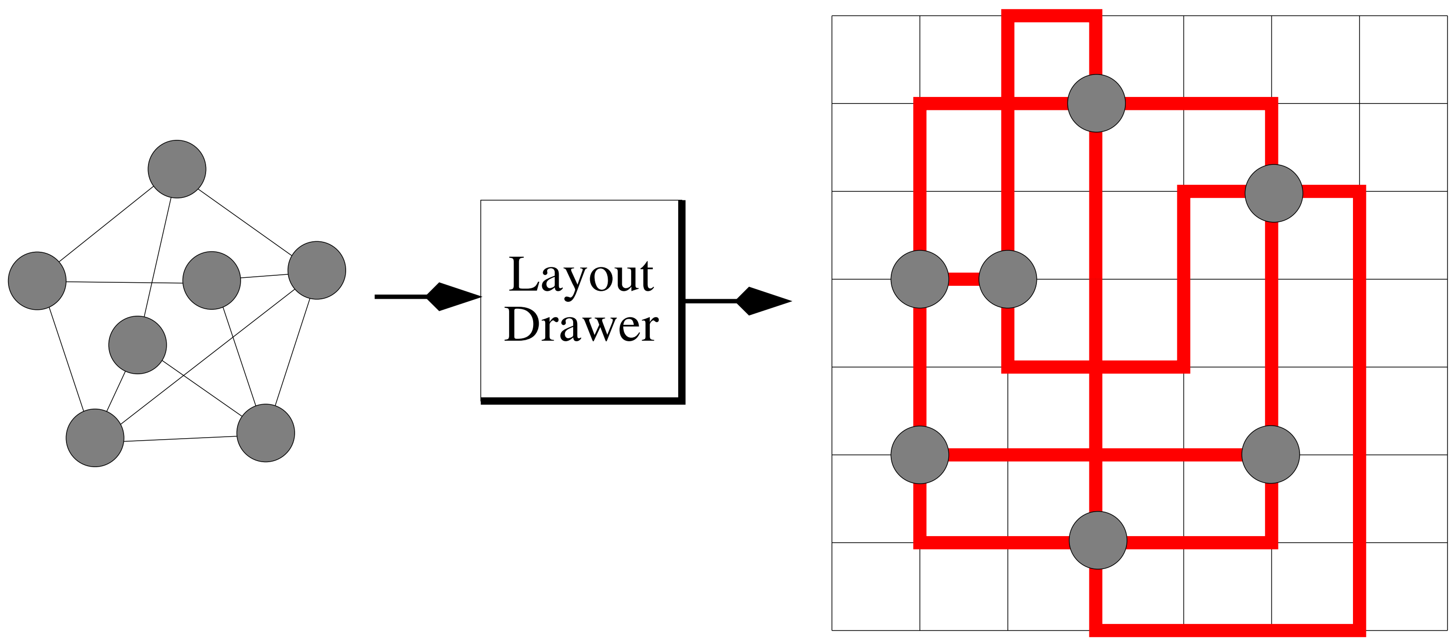
Dealing with the hardness of finding small area layouts



Bend the rules!

- Approximate the best layout [BL84, EGS03].

Dealing with the hardness of finding small area layouts



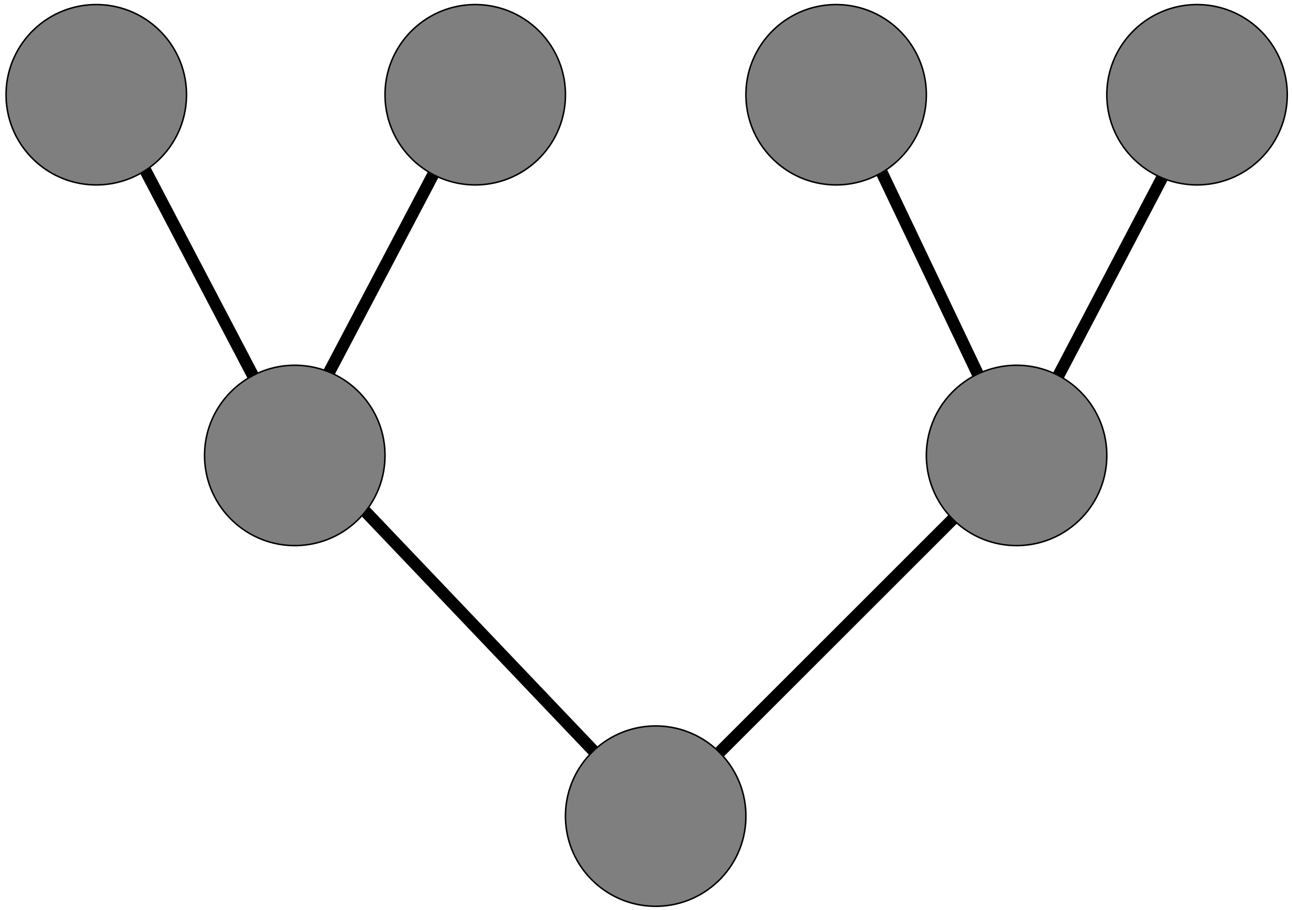
Bend the rules!

- Approximate the best layout [BL84, EGS03].
- Study layouts of specific “interesting” graphs.

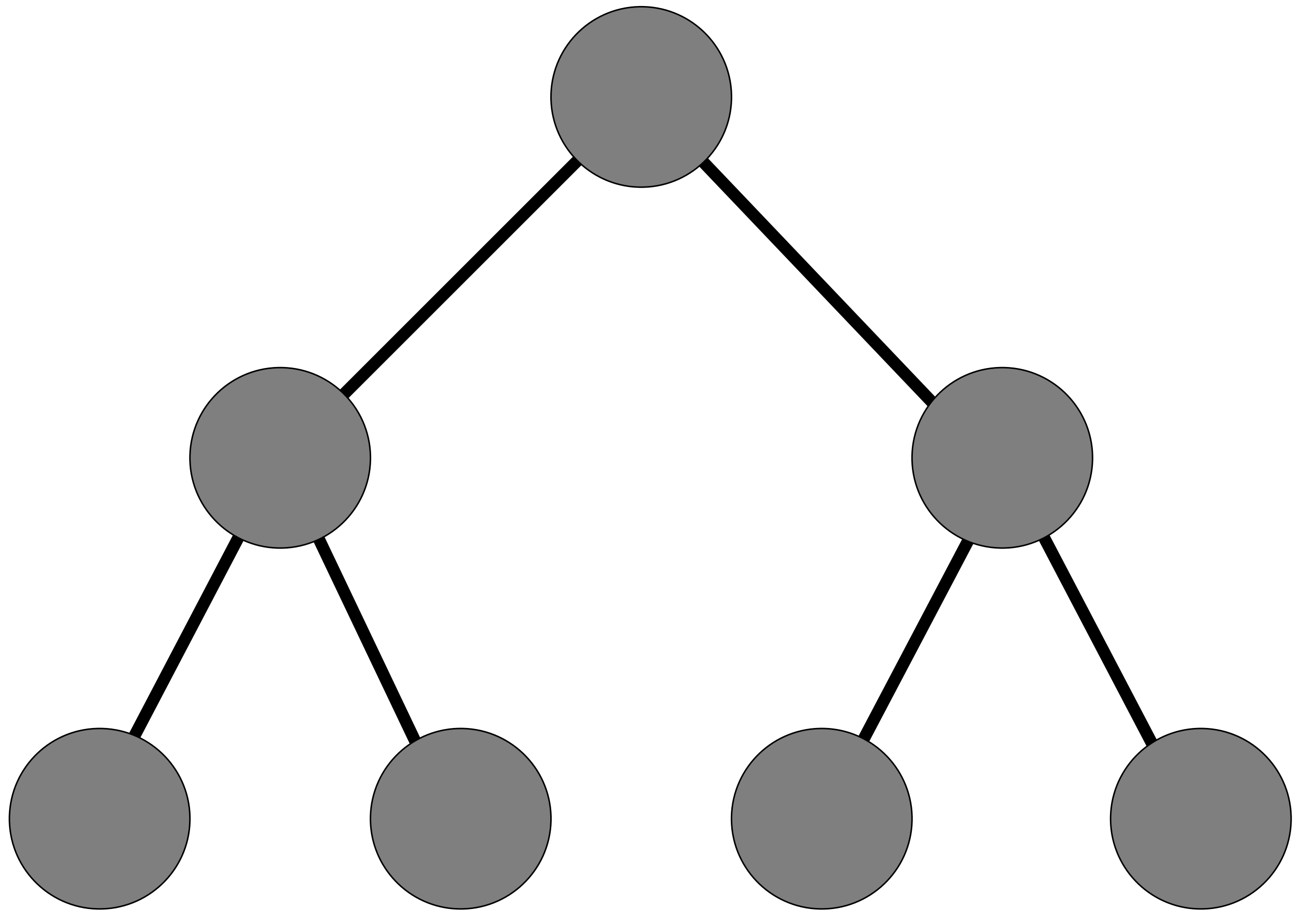
Some interesting graphs

- Complete binary tree.
- Butterfly (FFT, Omega)
- Mesh of Trees [Leighton 1983]

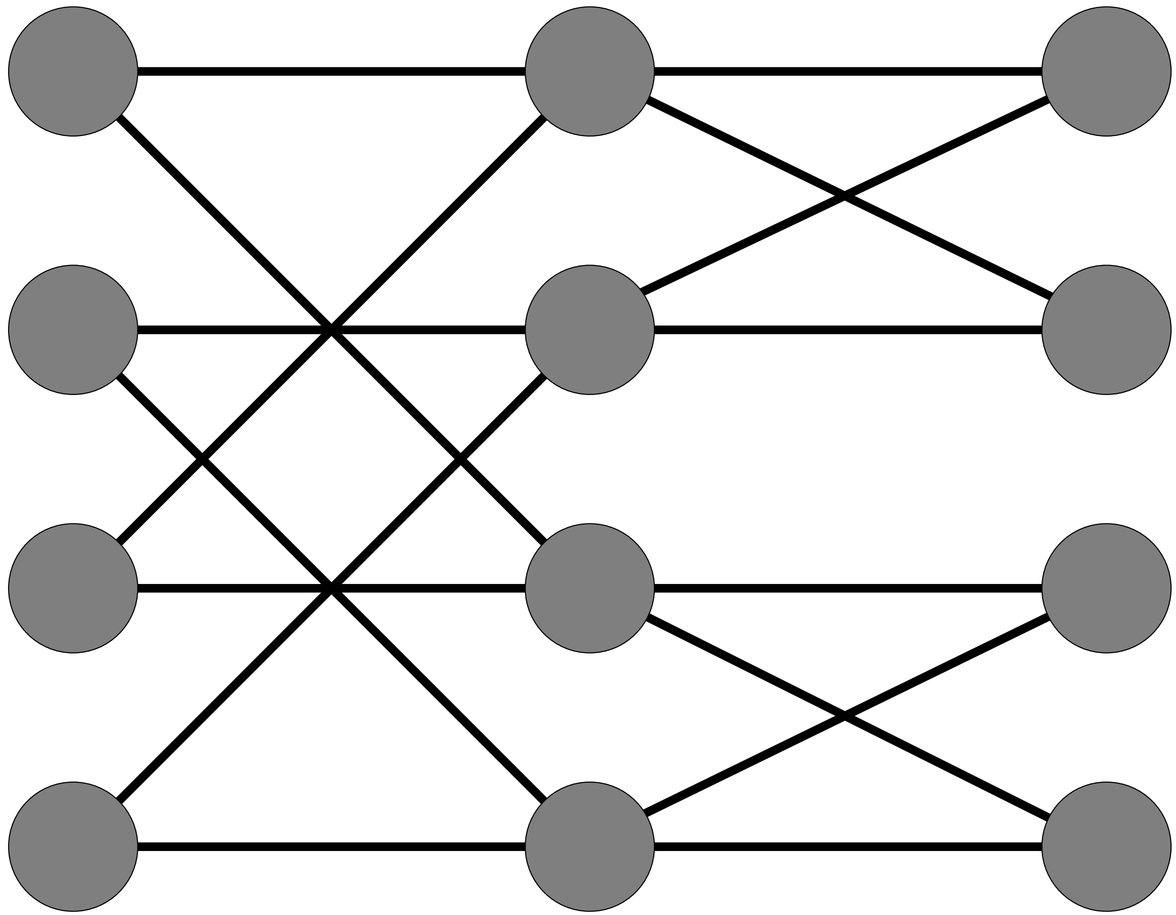
A complete binary tree



A complete binary tree



Butterfly



- Application: design of switches...
- Goal: find good layouts for Butterfly

Layered Cross Product [Even & Litman 1992]

- A structural decomposition of graphs

Layered Cross Product [Even & Litman 1992]

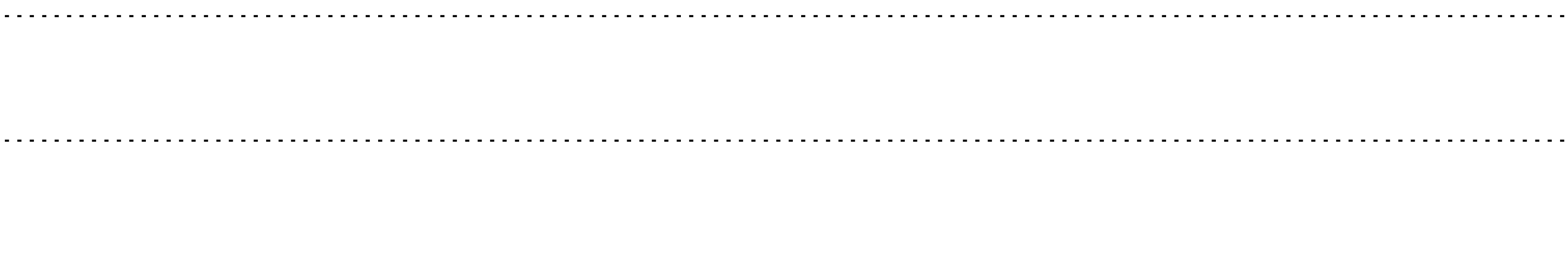
- A structural decomposition of graphs
- Simplifies proofs of graph properties

Layered Cross Product [Even & Litman 1992]

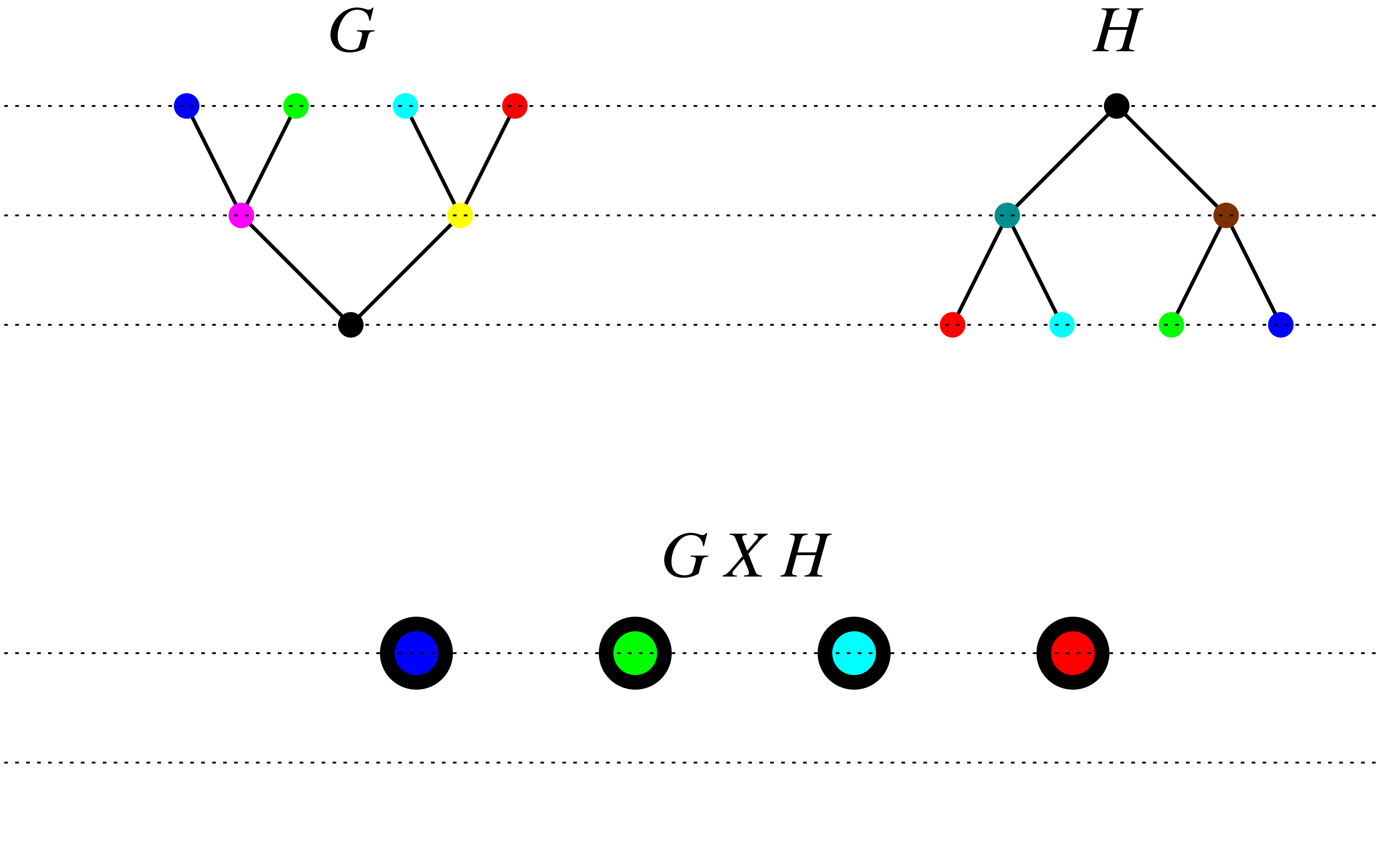
- A structural decomposition of graphs
- Simplifies proofs of graph properties
- Seems unrelated to layouts...

LCP : Butterfly

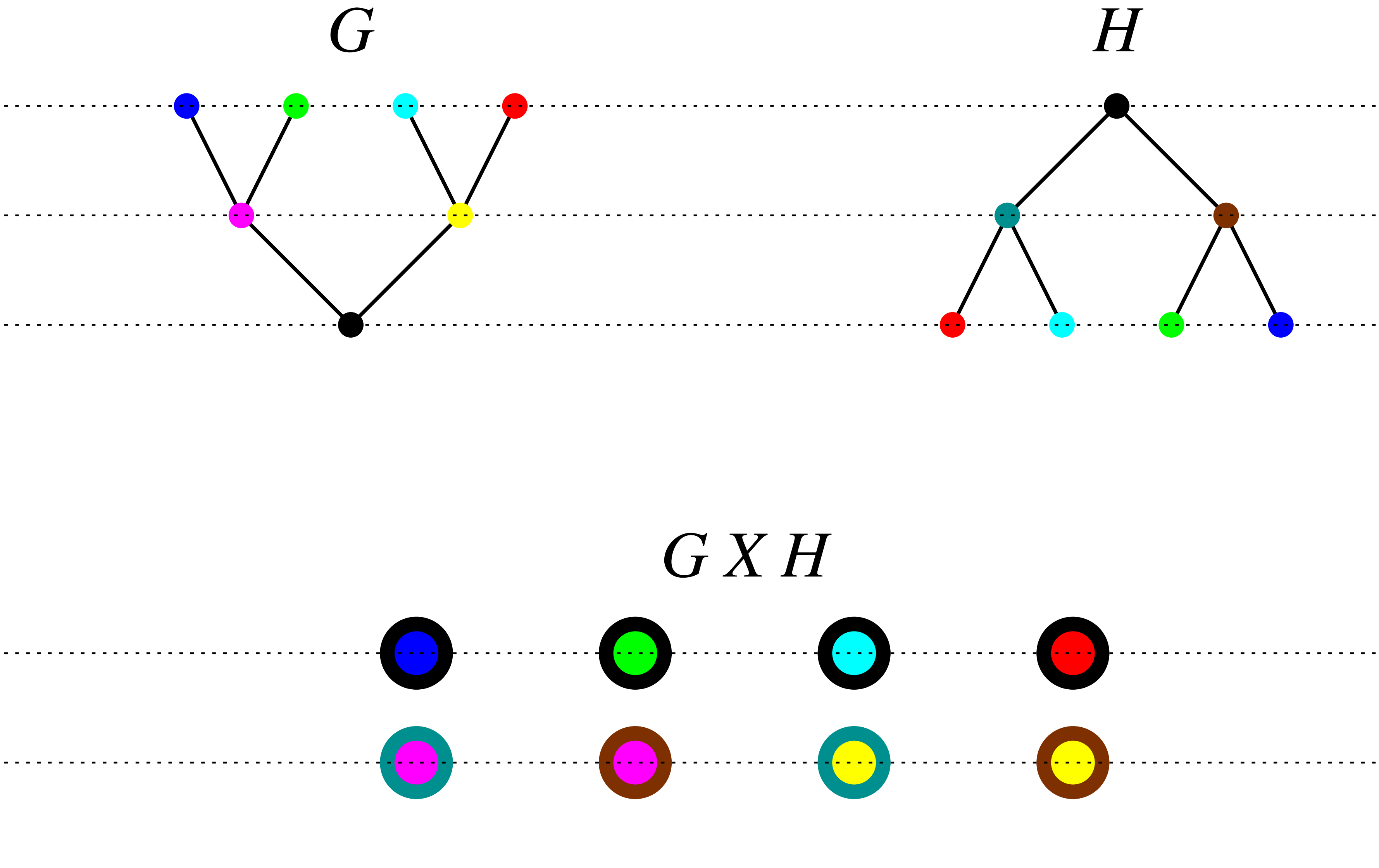
$G \, X \, H$



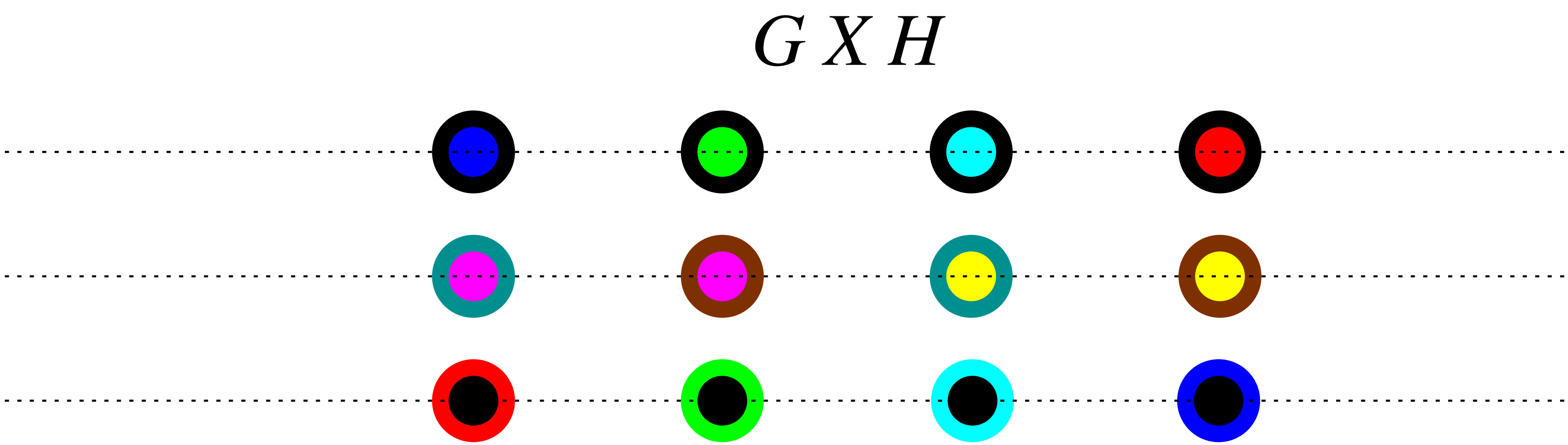
LCP : Butterfly



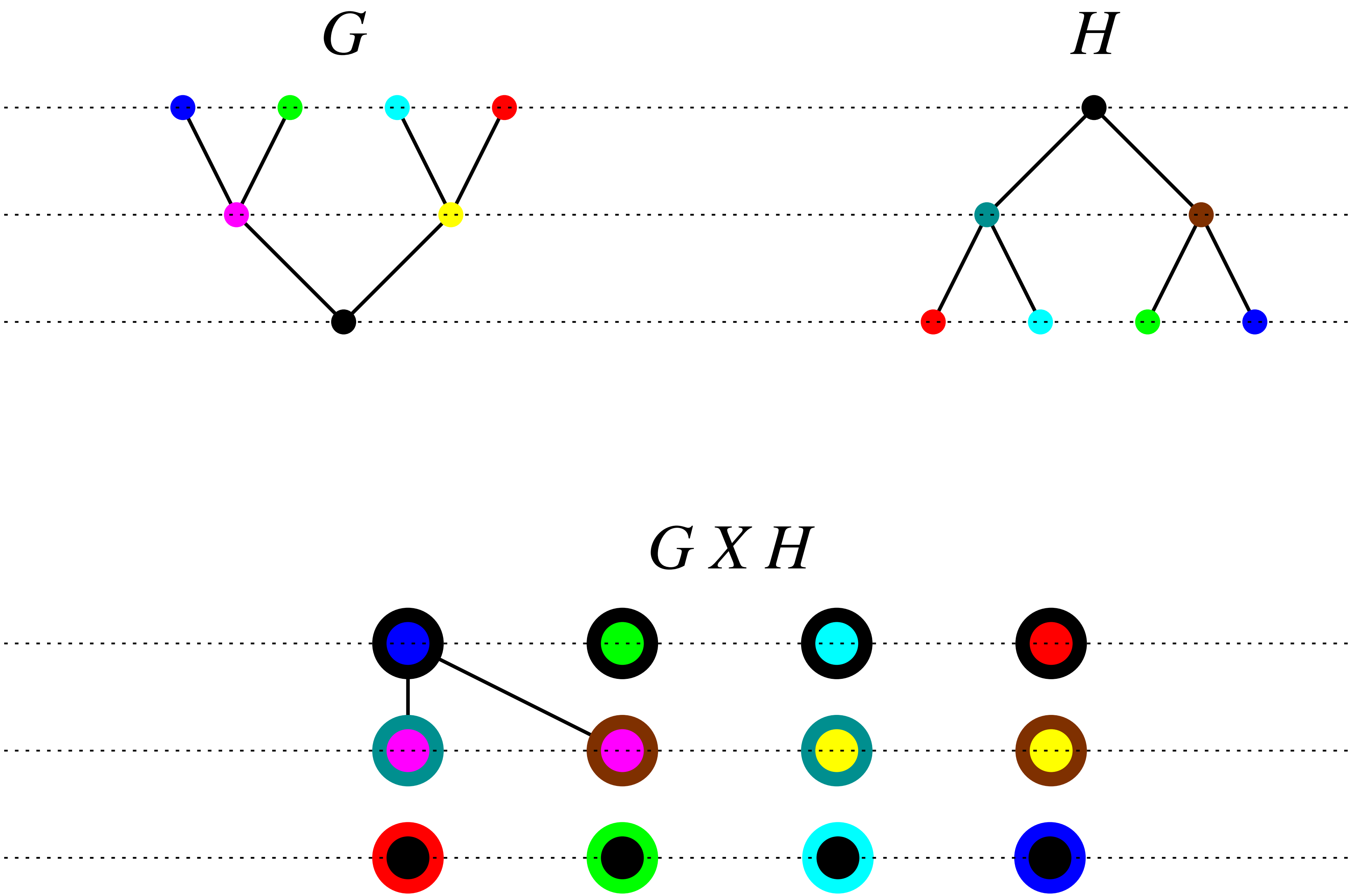
LCP : Butterfly



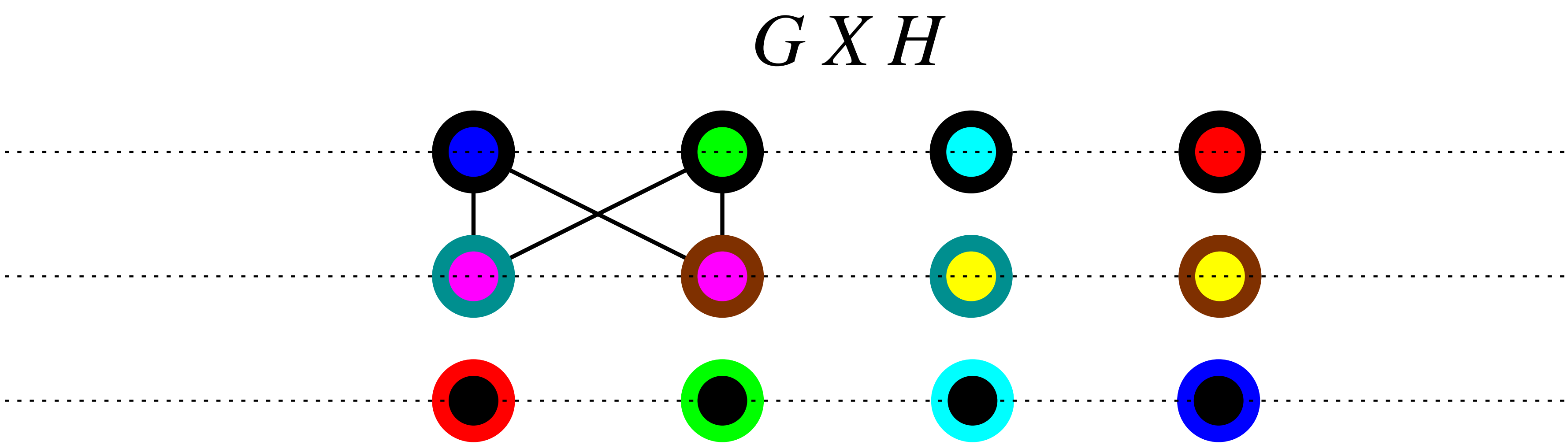
LCP : Butterfly



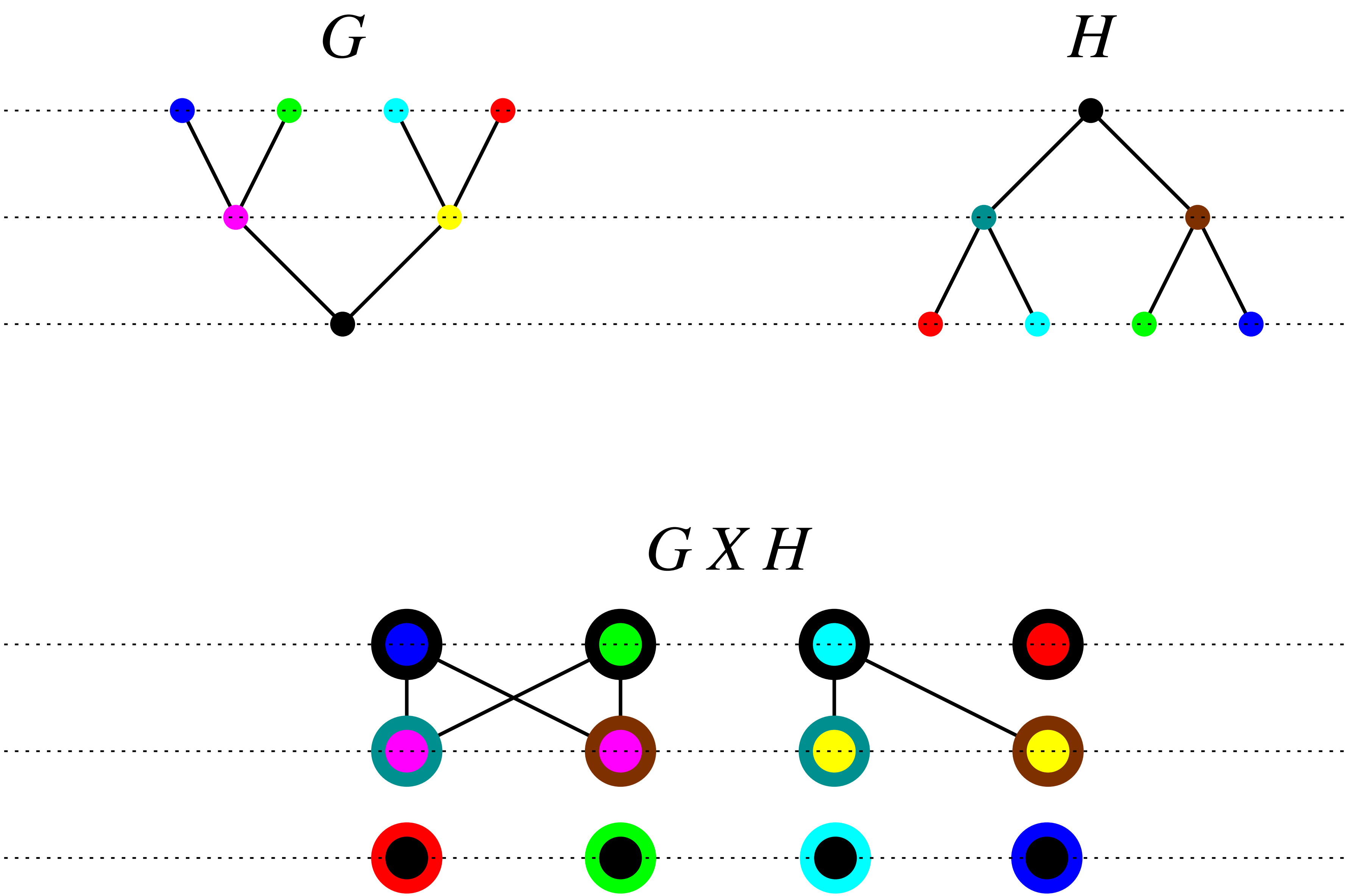
LCP : Butterfly



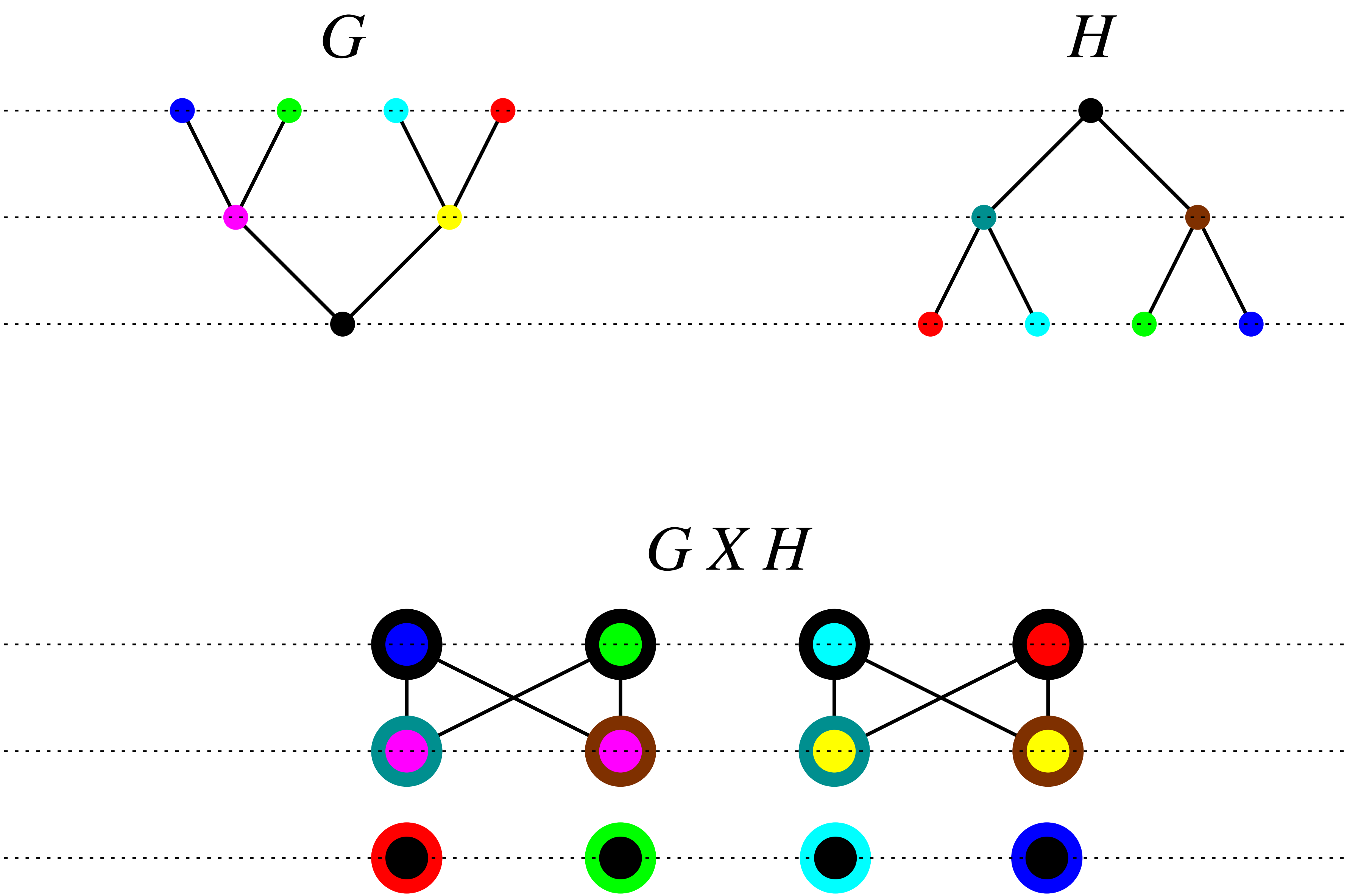
LCP : Butterfly



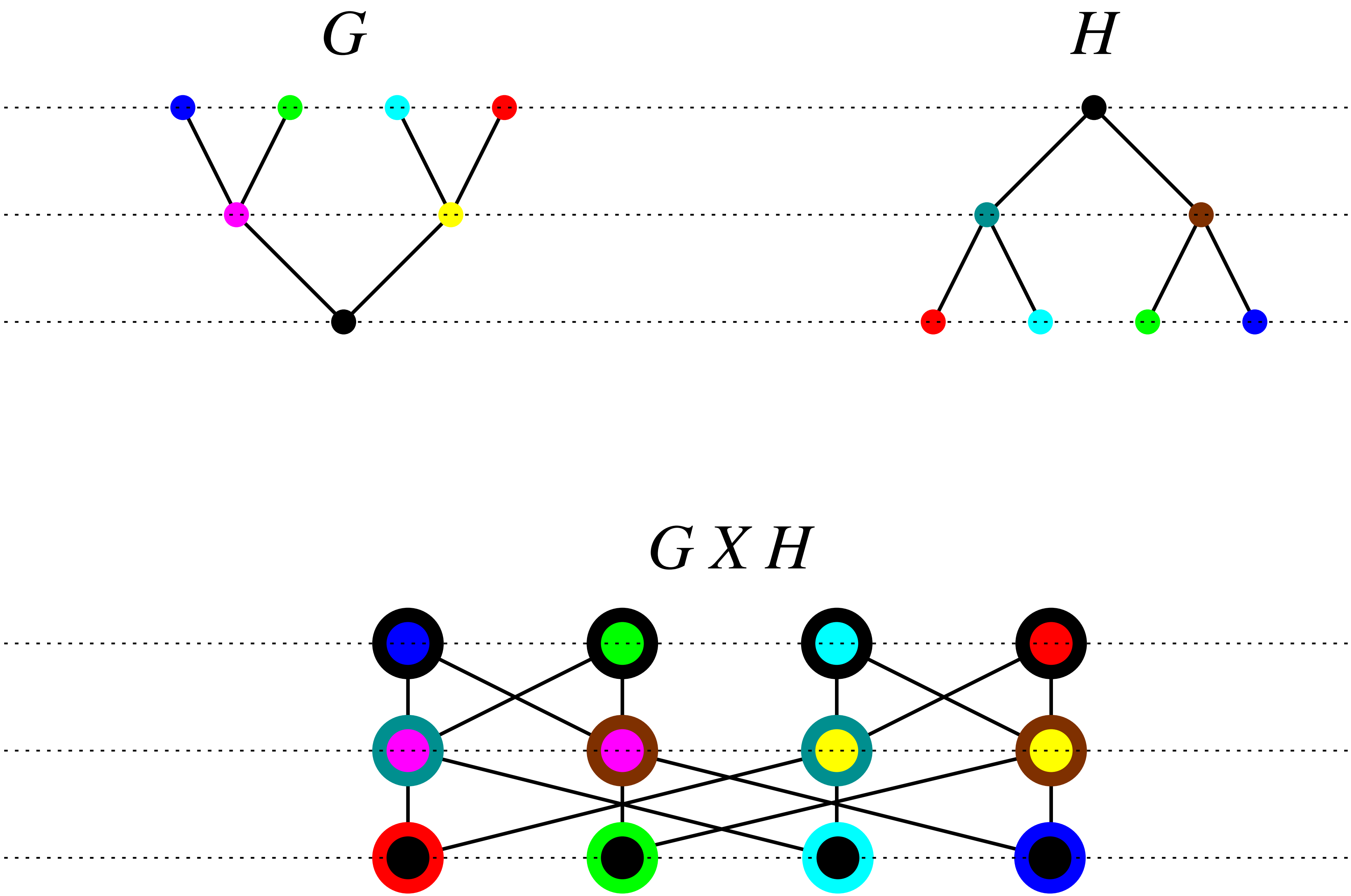
LCP : Butterfly



LCP : Butterfly



LCP : Butterfly

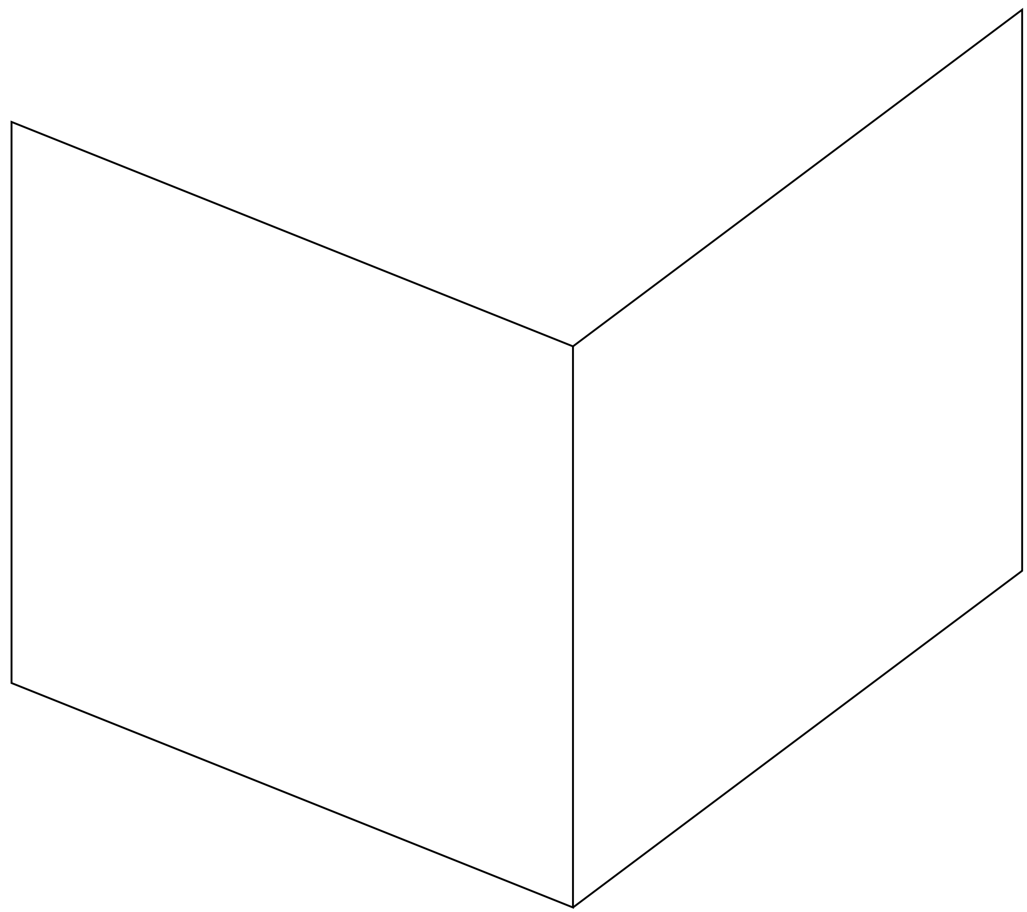


From LCP to Layouts

Projection Methodology [Even & Even 2000]

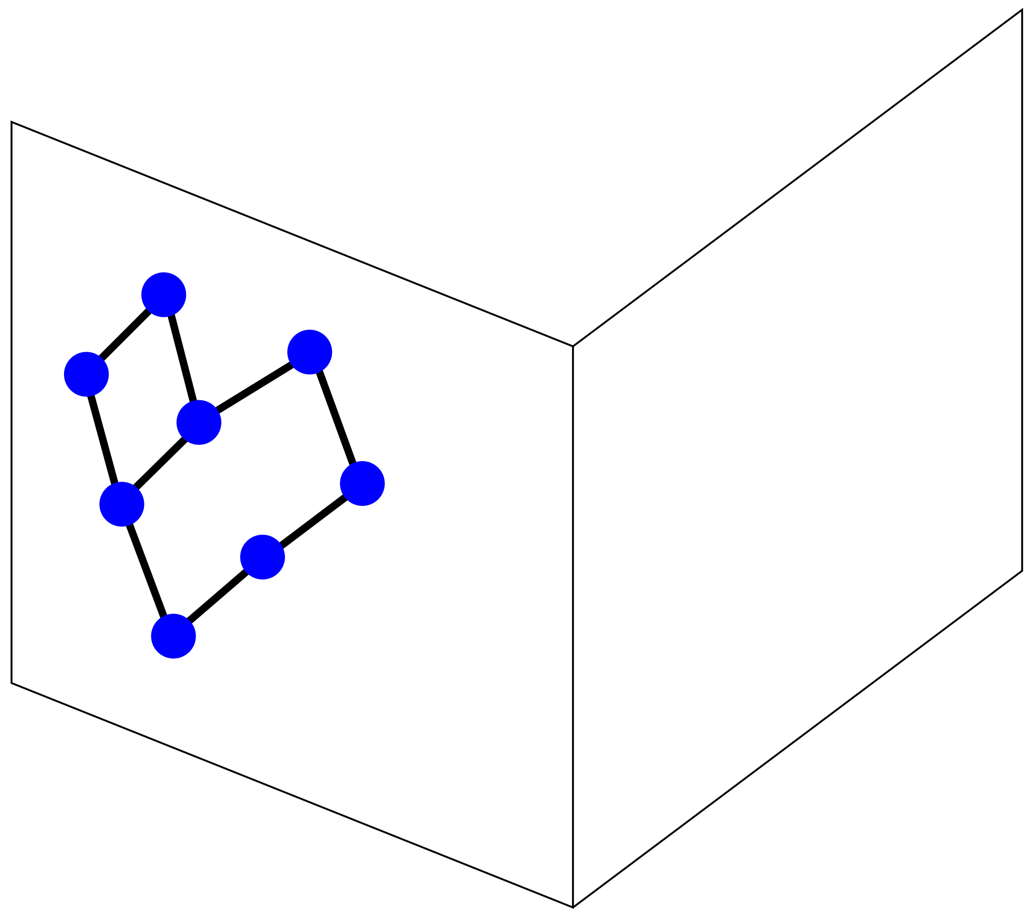
From LCP to Layouts

Projection Methodology [Even & Even 2000]



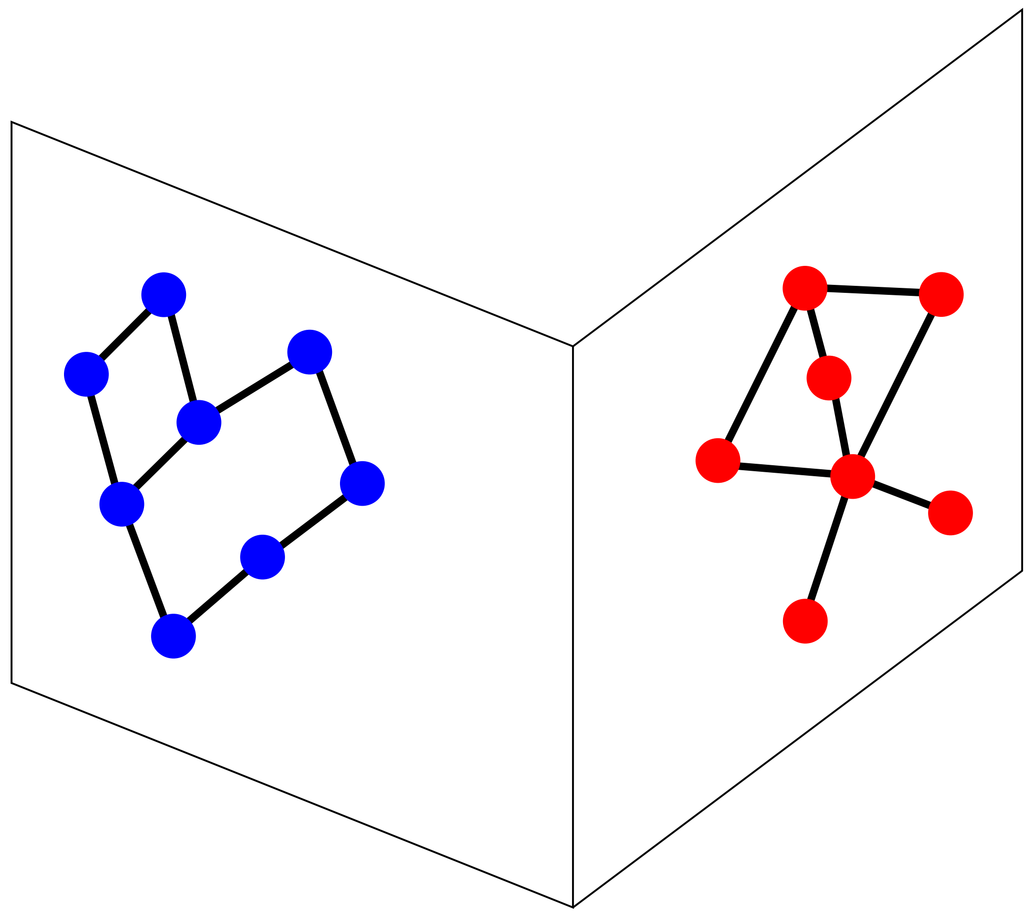
From LCP to Layouts

Projection Methodology [Even & Even 2000]



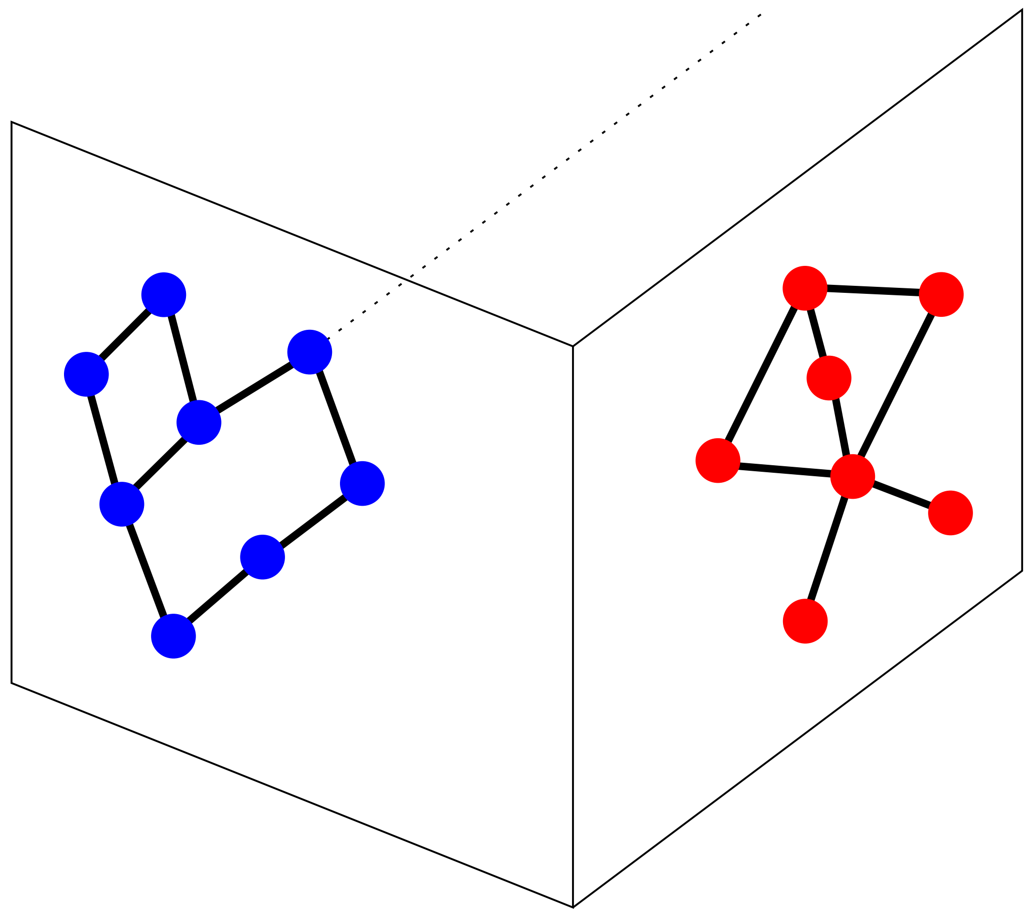
From LCP to Layouts

Projection Methodology [Even & Even 2000]



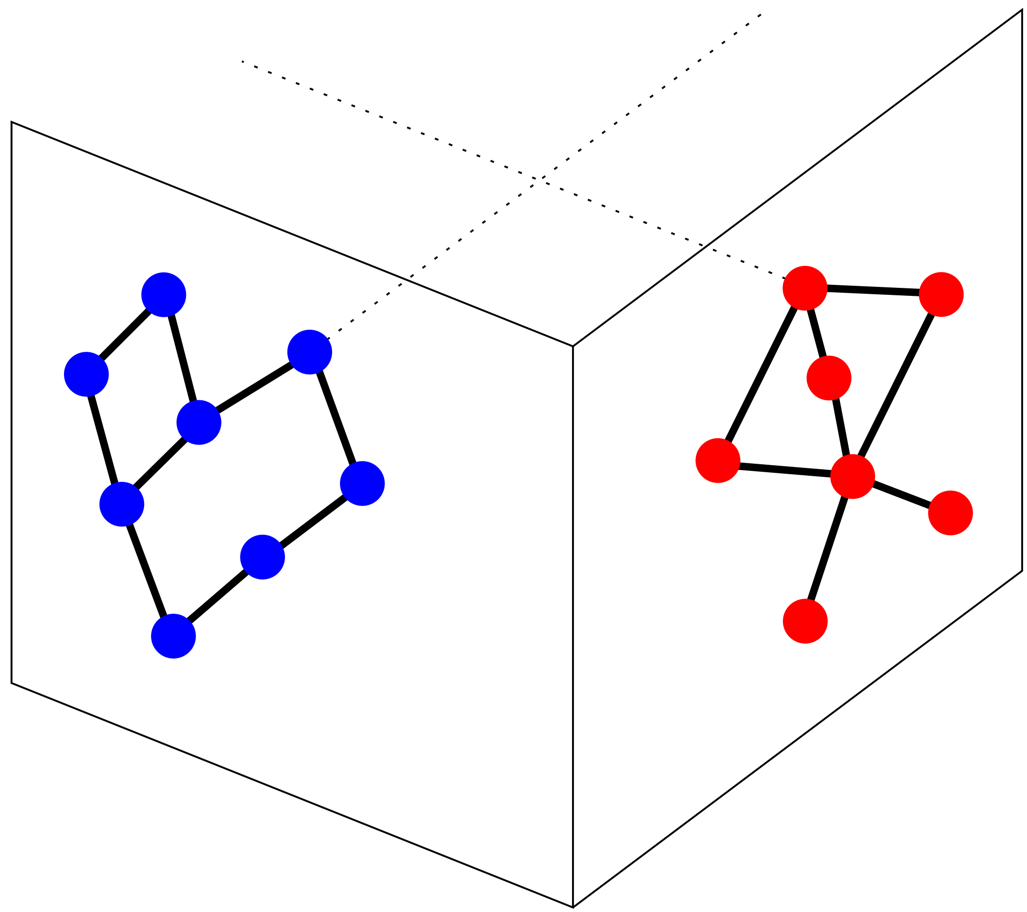
From LCP to Layouts

Projection Methodology [Even & Even 2000]



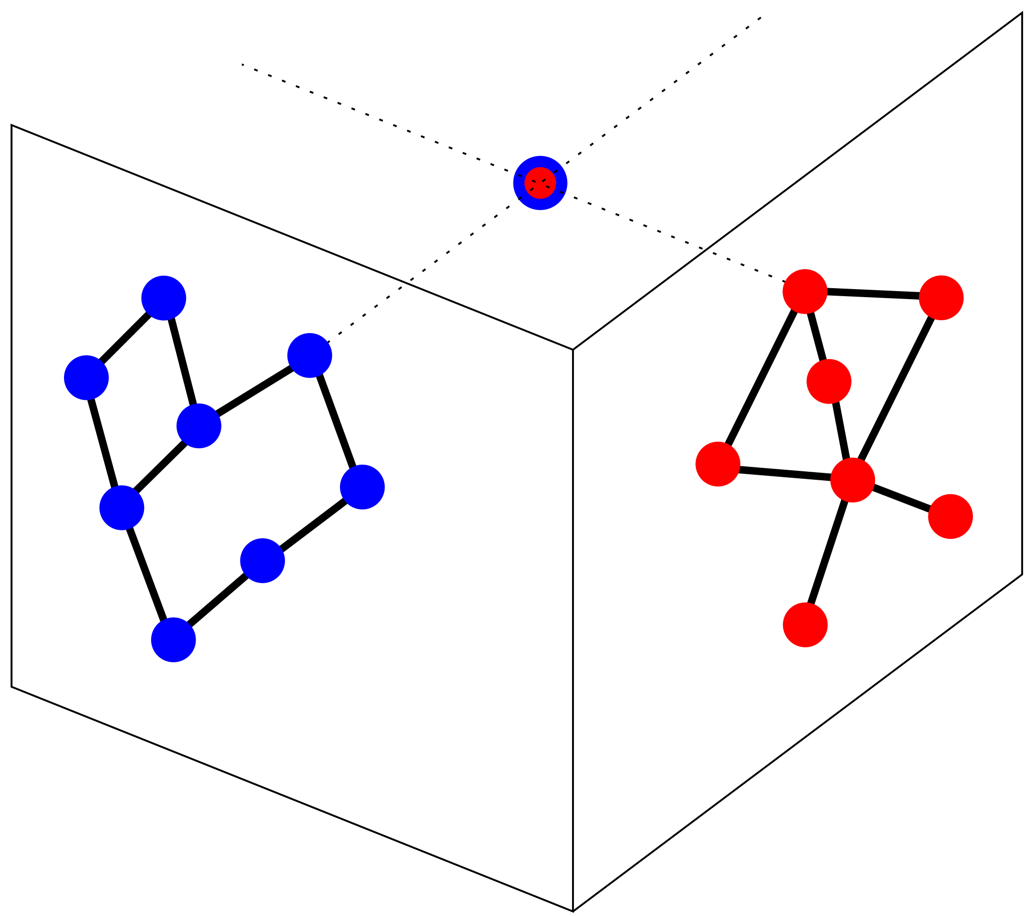
From LCP to Layouts

Projection Methodology [Even & Even 2000]

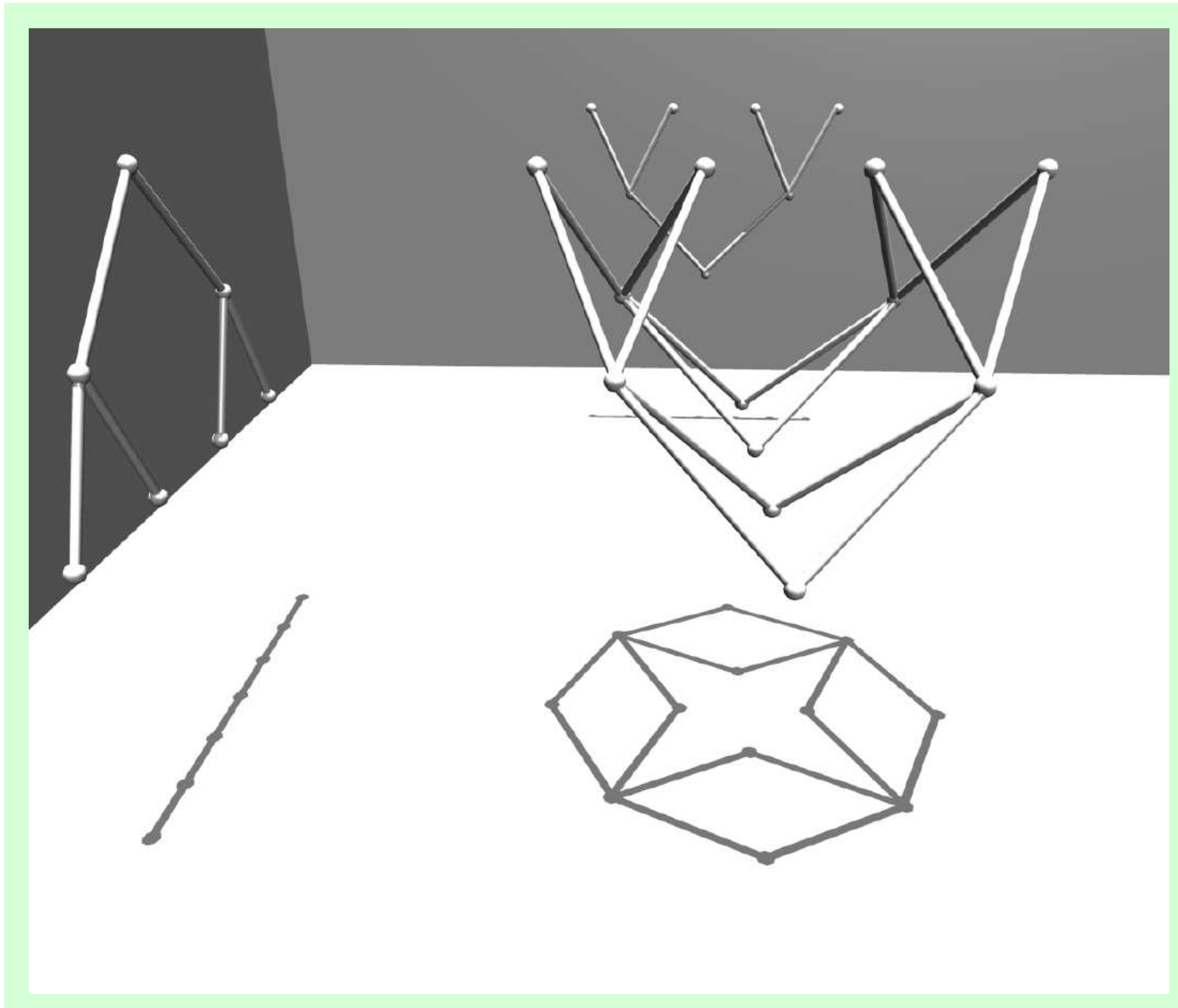


From LCP to Layouts

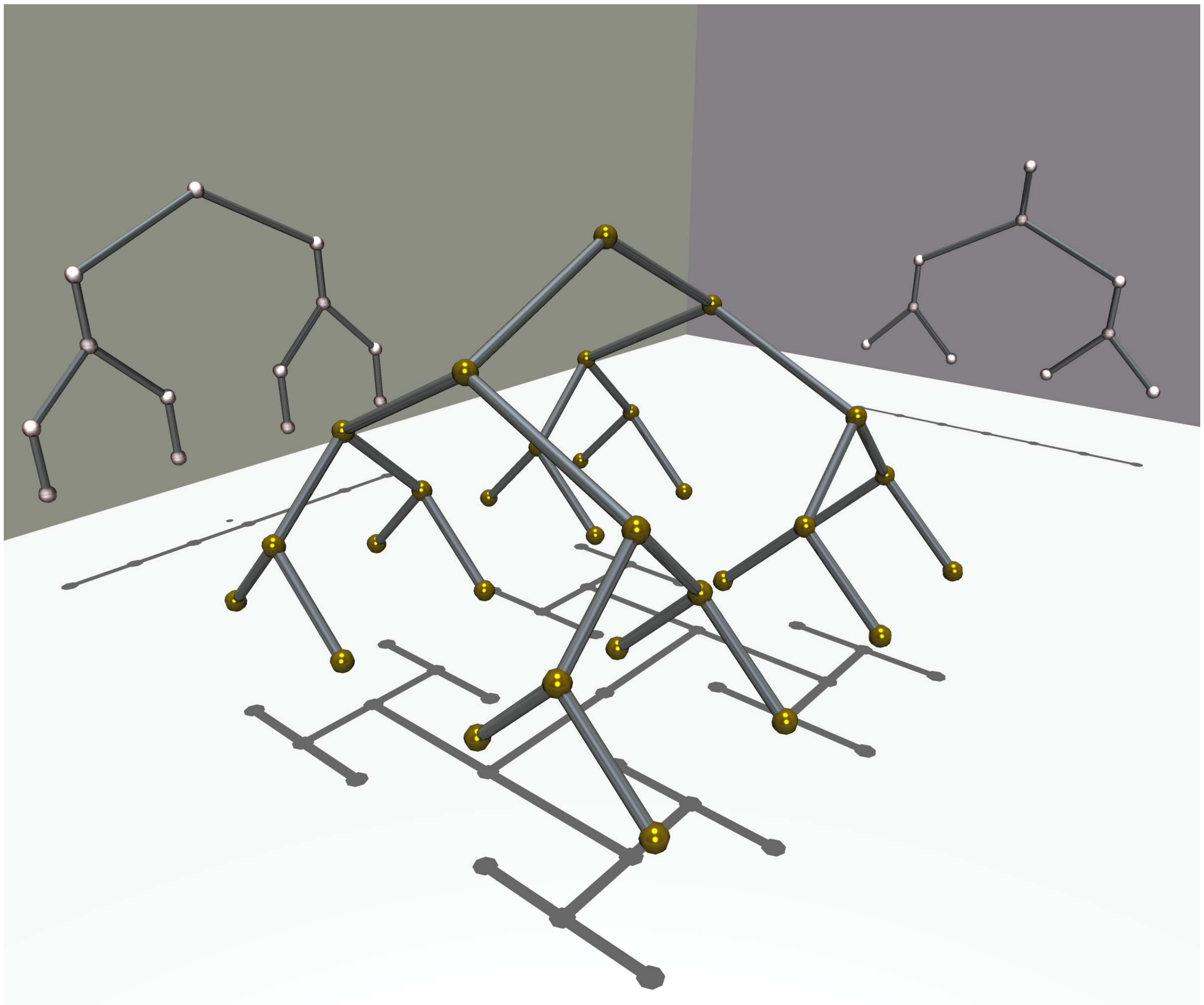
Projection Methodology [Even & Even 2000]



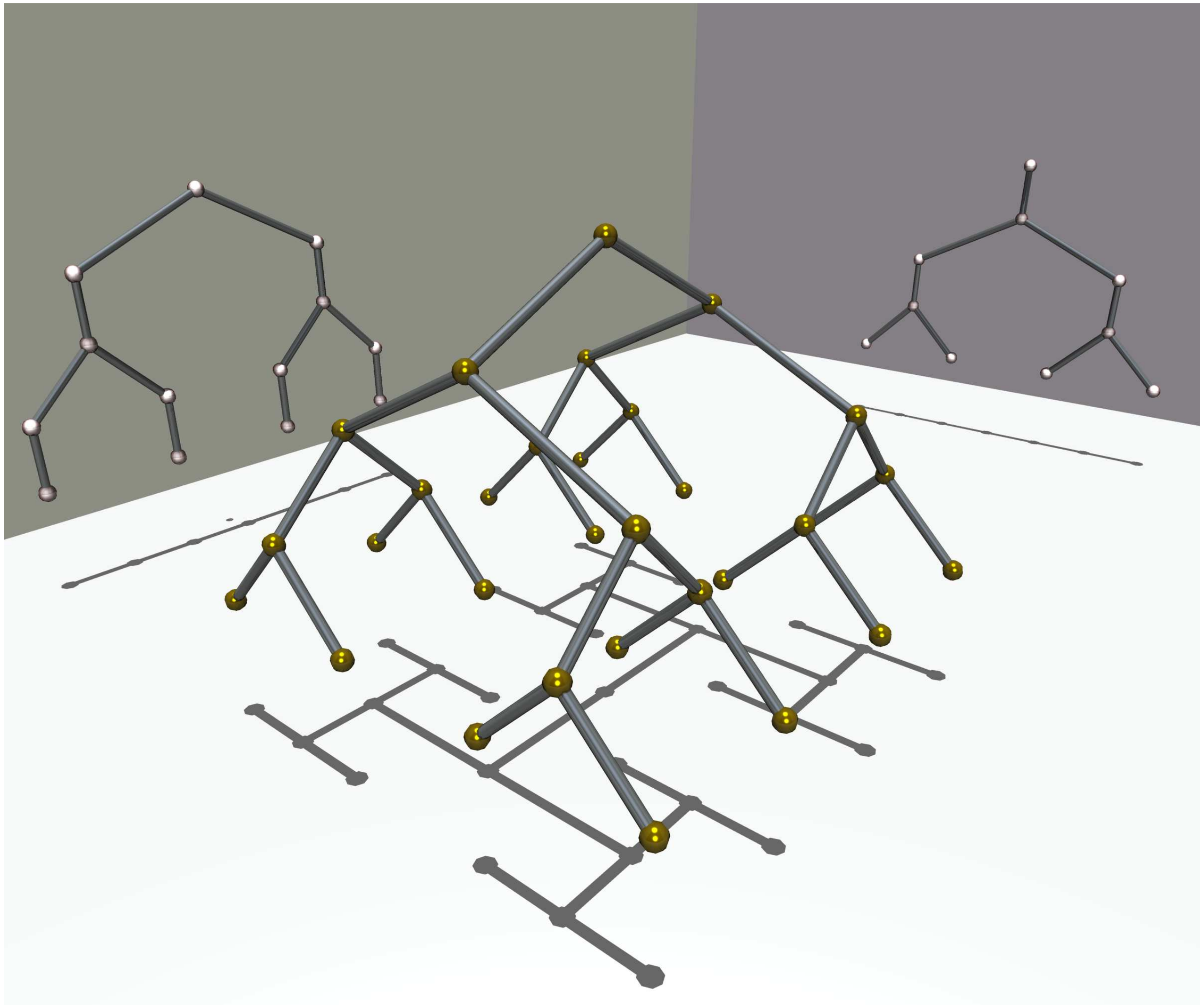
Projection Methodology with Butterfly



PM: getting rid of diagonal lines

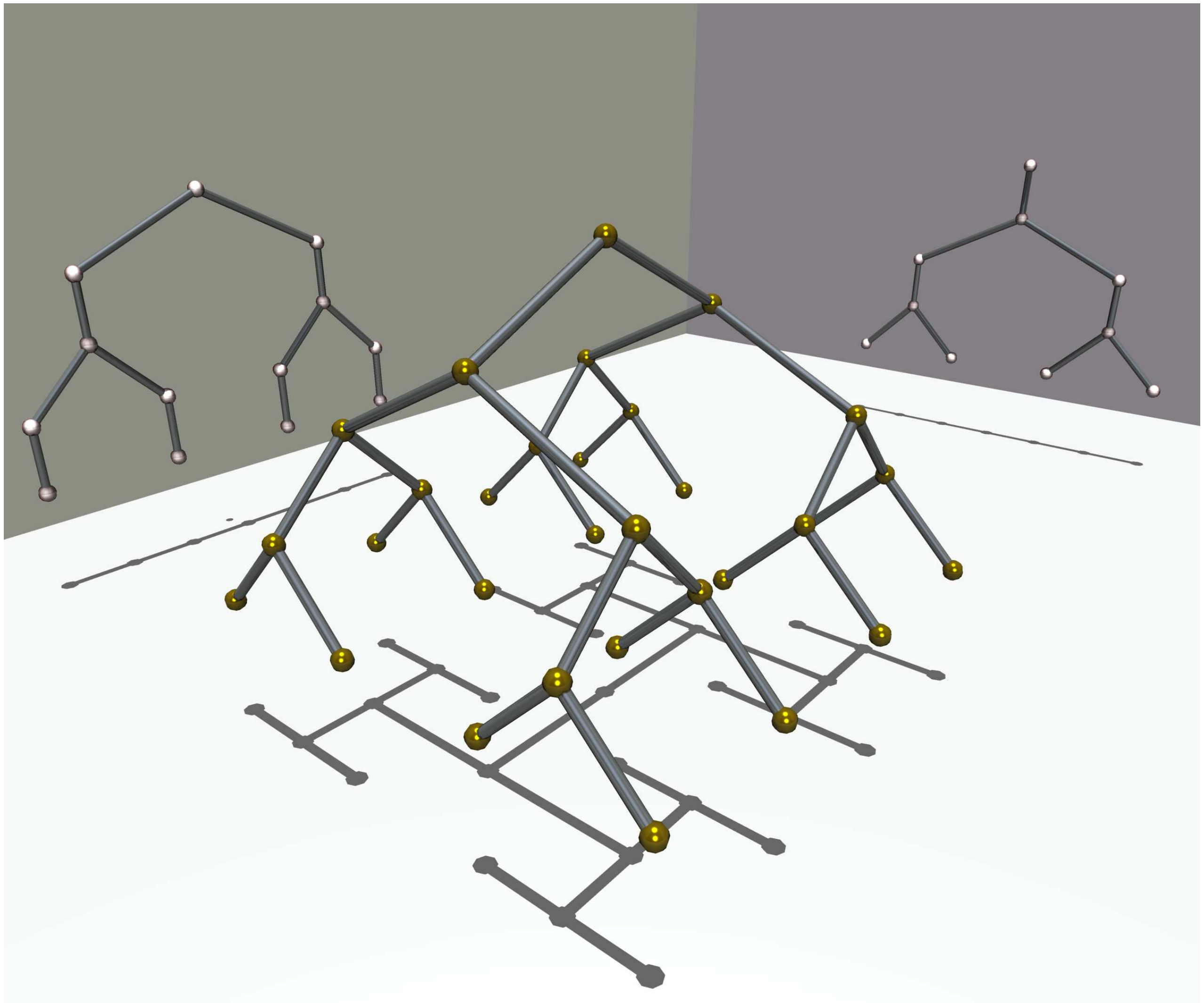


PM: getting rid of diagonal lines



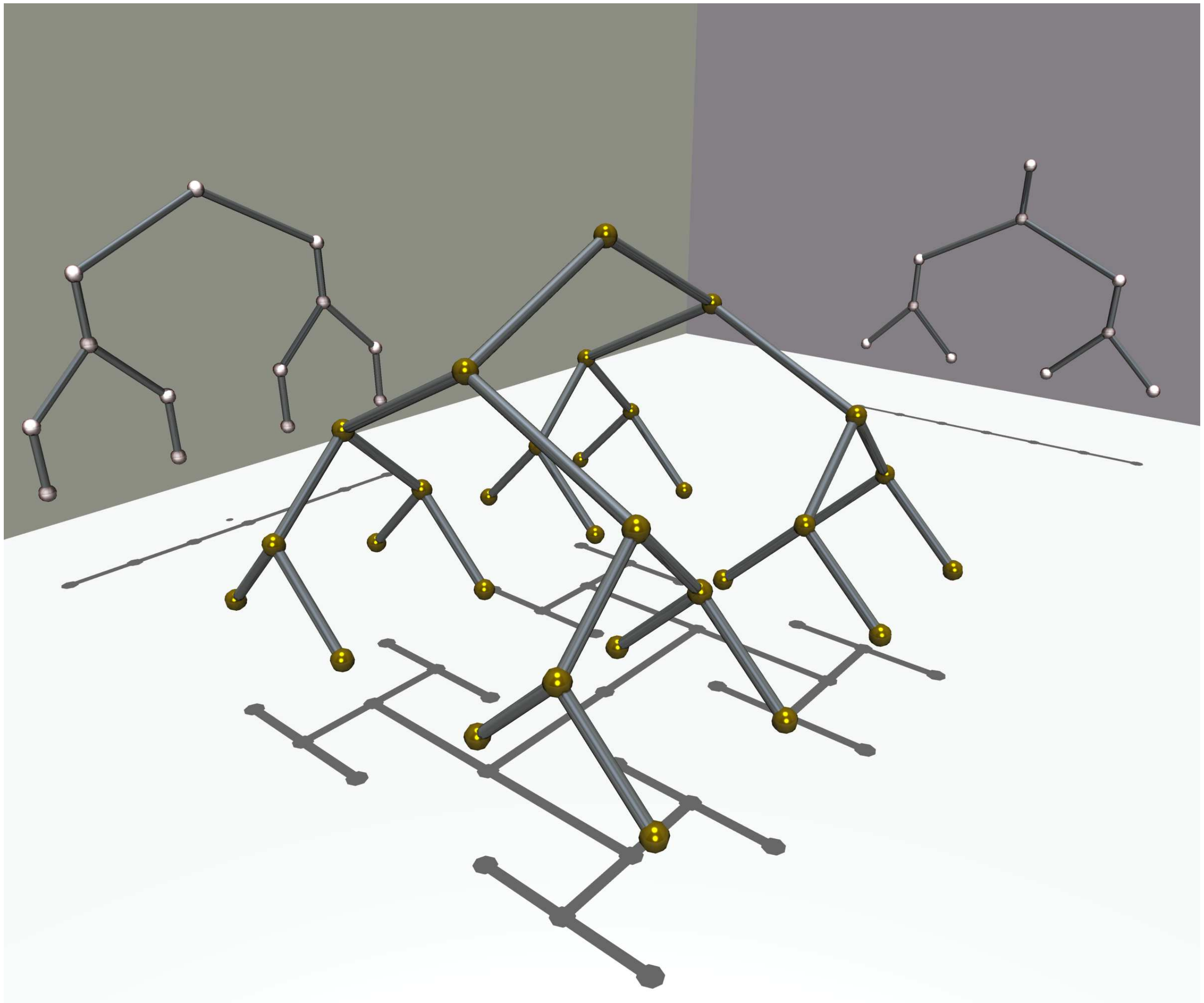
- Binary tree = LCP of trees.

PM: getting rid of diagonal lines



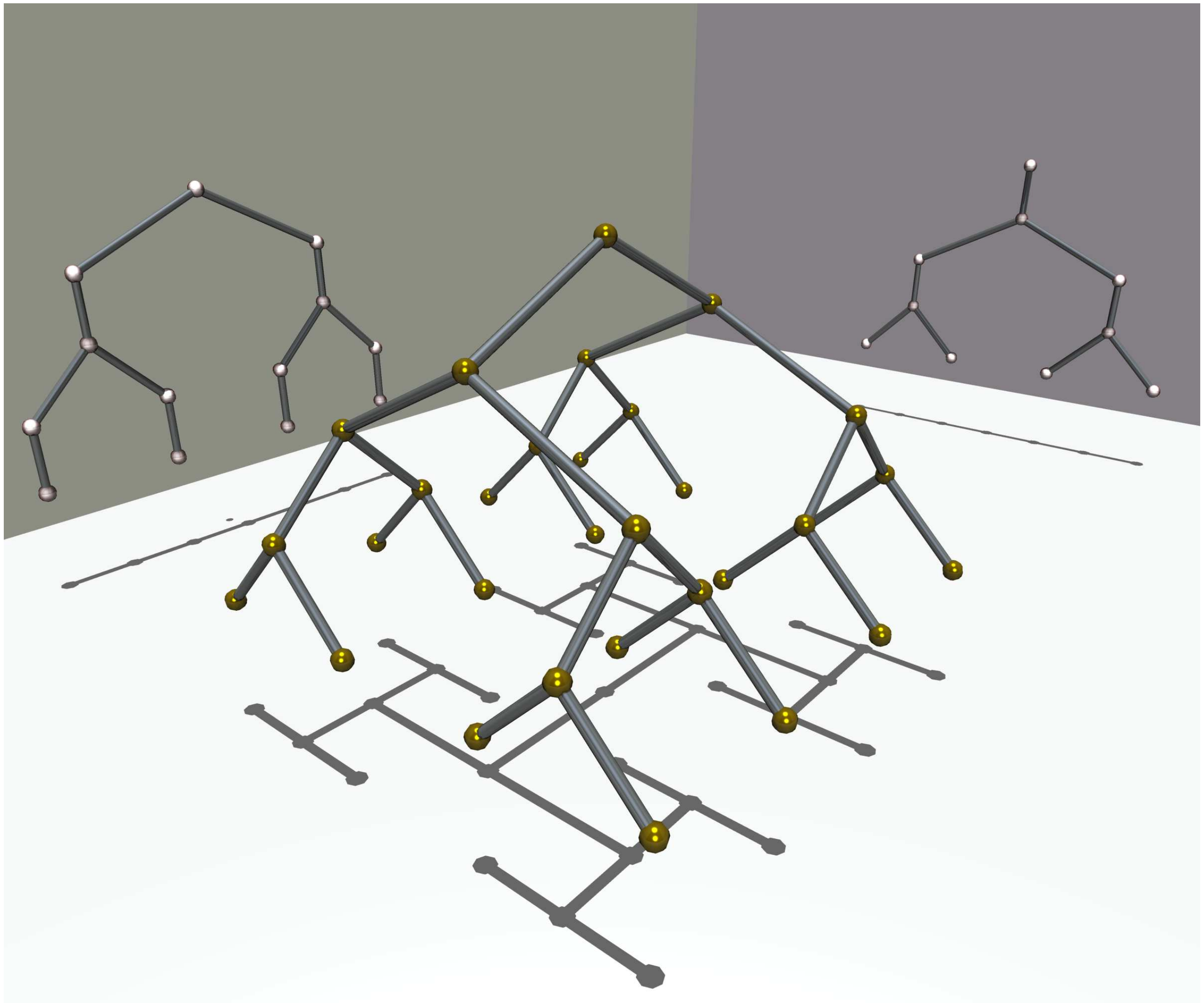
- Binary tree = LCP of trees.
- Alternate between "parallel" and "branching" levels.

PM: getting rid of diagonal lines



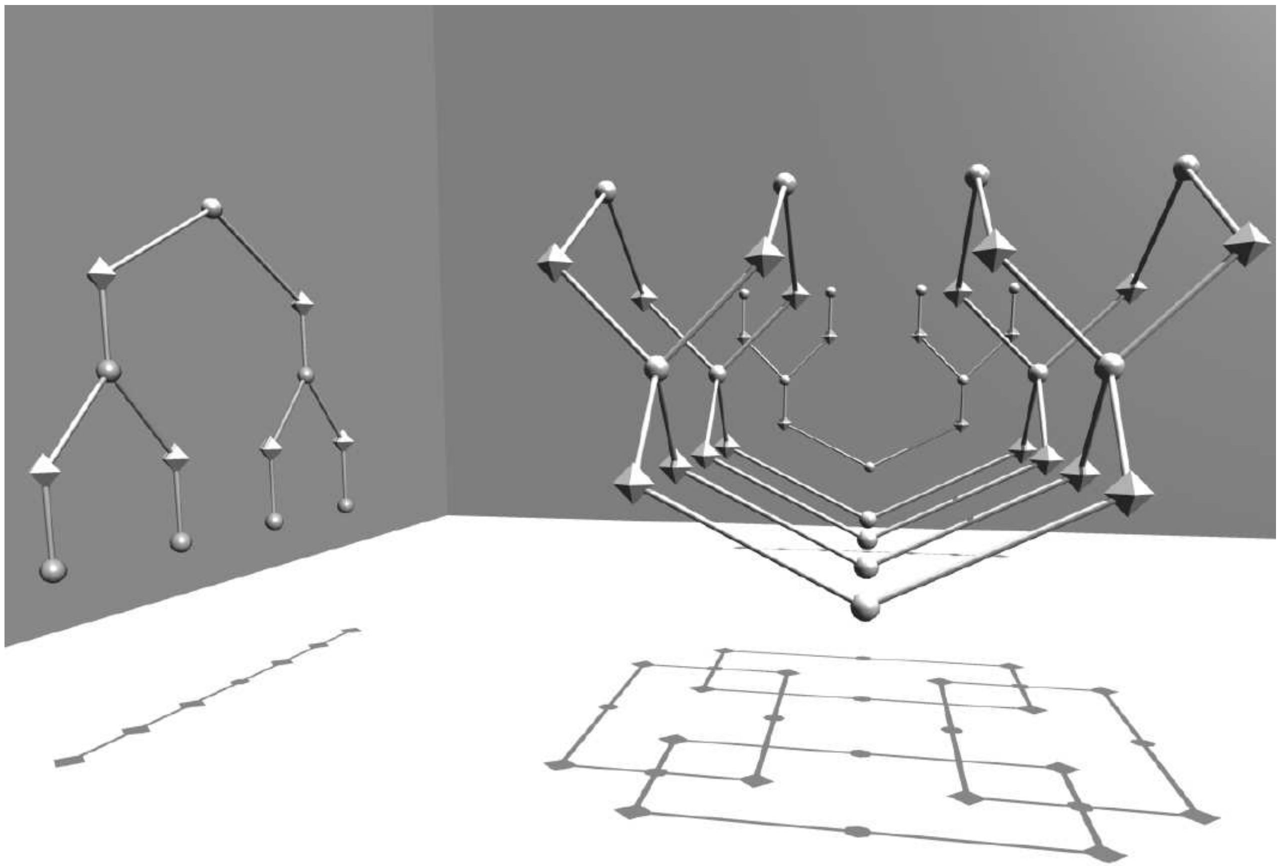
- Binary tree = LCP of trees.
- Alternate between “parallel” and “branching” levels.
- Projection is on grid lines

PM: getting rid of diagonal lines



- Binary tree = LCP of trees.
- Alternate between "parallel" and "branching" levels.
- Projection is on grid lines
- Yields H-tree layout of Shiloach!

PM gives new layout for Butterfly



What makes a work good?

- Motivated by a natural problem or an application (VLSI design)

What makes a work good?

- Motivated by a natural problem or an application (VLSI design)
- A clean abstraction (drawing rules of Shiloach)

What makes a work good?

- Motivated by a natural problem or an application (VLSI design)
- A clean abstraction (drawing rules of Shiloach)
- A mathematical characterization (LCP)

What makes a work good?

- Motivated by a natural problem or an application (VLSI design)
- A clean abstraction (drawing rules of Shiloach)
- A mathematical characterization (LCP)
- A method (PM)

What makes a work good?

- Motivated by a natural problem or an application (VLSI design)
- A clean abstraction (drawing rules of Shiloach)
- A mathematical characterization (LCP)
- A method (PM)
- A systematic explanation of previous results (H-trees)

What makes a work good?

- Motivated by a natural problem or an application (VLSI design)
- A clean abstraction (drawing rules of Shiloach)
- A mathematical characterization (LCP)
- A method (PM)
- A systematic explanation of previous results (H-trees)
- A new result (layouts for butterfly & MOT)

What makes a work good?

- Motivated by a natural problem or an application (VLSI design)
- A clean abstraction (drawing rules of Shiloach)
- A mathematical characterization (LCP)
- A method (PM)
- A systematic explanation of previous results (H-trees)
- A new result (layouts for butterfly & MOT)
- Bonus: geometry...