Pearls of Shimon Even

A talk in memory of my beloved father
The Academic College of Tel-Aviv Yaffo
June 3, 2004

crypto

PSPACE

NP

P

algs

graphs
Lots of great works

- Graph Algorithms: planarity testing, zero-one flows, connectivity, matching, dynamic algorithms.
- NP Completeness: timetables, integral multicommodity flows, ...
- PSPACE Completeness: Hex.
- Approximation Algorithms: vertex cover, local ratio.
- Cryptography: electronic wallet, digital signatures, signing contracts.
- Distributed Computation: synchronization, broadcast.
- Systolic Circuits: marked graphs, algorithms for retiming, Atrubin’s multiplier.
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Question: What should I talk about?
Layouts of graphs

- Wonderful topic
Layouts of graphs

- Wonderful topic
- Lots of pictures
Layouts of graphs

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- Work on layouts spanned 40 years
Layouts of graphs

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- Lots of pictures
- Work on layouts spanned 40 years
- ...we even have a joint paper about it!
Graphs

Consider a set of 5 people:

- Avi knows Benny: (A; B)
- Benny knows Cindy: (B; C) and also (C; D), (D; E), (E; A).
Consider a set of 5 people:

Avi, Benny, Cindy, Danny, Ella.
Graphs

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- Write down the pairs of people that know each other:
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**Graph** = set of people and list of pairs of people.
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- **graph** = set of people and list of pairs of people.
- Other examples: cities & roads, electrical components & wires, etc.
Depiction of graphs

Consider the graph with

- People: Avi, Benny, Cindy, Danny, Ella
- Pairs of people: $(A, B), (B, C), (C, D), (D, E), (E, A)$.

Good idea to draw it:
Depiction of graphs

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- People: Avi, Benny, Cindy, Danny, Ella
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Good idea to draw it:

A

B

C

D

E
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Good idea to draw it:
Different ways to depict graphs

The same cycle on 5 vertices can be drawn in many ways:

\[ A \rightarrow B \rightarrow C \rightarrow D \rightarrow E \rightarrow A \]

Question: Which drawing is "better"?

Answer: Prefer drawings without crossings of edges.
Different ways to depict graphs

The same cycle on 5 vertices can be drawn in many ways:

A

E  B

D  C
Different ways to depict graphs

The same cycle on 5 vertices can be drawn in many ways:

- \( \text{A-B-C-D-E} \)
- \( \text{B-A-E-D-C} \)

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Different ways to depict graphs

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DEF: A graph is planar if it can be drawn so that no two edges cross.
**Planar Graphs**

**DEF:** A graph is **planar** if it can be drawn so that no two edges cross.

![Planar Graph](image-url)
DEF: A graph is **planar** if it can be drawn so that no two edges cross.

- A planar graph
- A non-planar graph
Characterization of planar graphs

**THM:** [Kuratowski 1930] A graph is planar if and only if it does not contain a “copy” of $K_5$ or $K_{3,3}$. 
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$K_{3,3}$ and $K_5$ are the “smallest” non-planar graphs and must “hide” in every non-planar graph. Often called forbidden graphs.
Is this graph planar?

This is not a planar drawing - but does a planar drawing exist? Look for a copy of forbidden graph \((K_{3,3})\)...
Is this graph planar?

We mark the “red” nodes

We found a “copy” of $K_3 \cong K_3$; therefore, the graph is non-planar!
Is this graph planar?

We mark the “blue” nodes

We found a $K_3^3$; therefore, the graph is non-planar!
Is this graph planar?

Mark paths from the 1st red node to the blue nodes

We found a "copy" of $K_3^3 = \text{non-planar}!$
Is this graph planar?

Mark paths from the 2nd red node to the blue nodes

We found a "copy" of $K_3^3$; $K_3^3$ is non-planar!
Is this graph planar?

Mark paths from the 3rd red node to the blue nodes

We found a "copy" of $K_3^3 = \text{non-planar!}$
Is this graph planar?

Mark paths from the 3rd red node to the blue nodes. We found a “copy” of $K_{3,3}$ → non-planar!
Planarity Testing

An algorithm for planarity testing:
Planarity Testing

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Planarity Tester

- yes: A planar drawing
- no: A "copy" of $K_5$ or $K_{2,3}$
Planarity Testing

An algorithm for planarity testing:

[Lempel, Even, & Cederbaum 1967]:
A polynomial time algorithm for planarity testing. Linear time realizations by [Even & Tarjan 76, Booth & Lueker 76].
Why investigate graph drawing?

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→ Want to find drawings with smallest possible area...
Printed Circuit Boards
Printed Circuit Boards
Layout of graph

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Question: Do planar drawings of planar graphs lead to small area layout?
Example: [Shiloach] Nested triangles...
Planarity & Layout

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- every additional triangle adds linear area to drawing
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\[ \text{area} = n^2 \]
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- $\rightarrow$ area $= n^2$
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- every additional triangle adds linear area to drawing
- $\implies$ area = $n^2$

- non-planar drawing
- every additional triangle adds constant area to drawing
Planarity & Layout

Question: Do planar drawings of planar graphs lead to small area layout?

Example: [Shiloach] Nested triangles...

- Planar graph & drawing
  - Every additional triangle adds linear area to drawing
  - $\text{area} = n^2$

- Non-planar drawing
  - Every additional triangle adds constant area to drawing
  - $\text{area} = n$
Conclusion: Using planar drawings is not always a good idea... Layout problem is much harder!
Dealing with the hardness of finding small area layouts
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Bend the rules!
Dealing with the hardness of finding small area layouts

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- Approximate the best layout [BL84, EGS03].
Dealing with the hardness of finding small area layouts

Bend the rules!

- Approximate the best layout [BL84, EGS03].
- Study layouts of specific “interesting” graphs.
Some interesting graphs

- Complete binary tree.
- Butterfly (FFT, Omega)
- Mesh of Trees [Leighton 1983]
A complete binary tree
A complete binary tree
Butterfly

- Application: design of switches...
- Goal: find good layouts for Butterfly
Layered Cross Product [Even & Litman 1992]

- A structural decomposition of graphs
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- Simplifies proofs of graph properties
Layered Cross Product [Even & Litman 1992]

- A structural decomposition of graphs
- Simplifies proofs of graph properties
- Seems unrelated to layouts...
LCP : Butterfly

\[ G \times H \]
LCP : Butterfly

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\[ G \]  \hspace{2cm}  \[ H \]

\[ G \times H \]
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$G \times H$

Diagram of graphs $G$, $H$, and their Cartesian product $G \times H$. The vertices and edges are color-coded to represent different sets or types in the graph theory context.
LCP : Butterfly

\[ G \]

\[ H \]

\[ G \times H \]
LCP : Butterfly

\[ G \]

\[ H \]

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$G \times H$
From LCP to Layouts

Projection Methodology [Even & Even 2000]
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From LCP to Layouts

Projection Methodology [Even & Even 2000]
Projection Methodology with Butterfly
PM: getting rid of diagonal lines
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- Binary tree = LCP of trees.
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Alternate between “parallel” and “branching” levels.
PM: getting rid of diagonal lines

- Binary tree = LCP of trees.
- Alternate between “parallel” and “branching” levels.
- Projection is on grid lines.
PM: getting rid of diagonal lines

- Binary tree = LCP of trees.
- Alternate between “parallel” and “branching” levels.
- Projection is on grid lines
- Yields H-tree layout of Shiloach!
PM gives new layout for Butterfly
What makes a work good?

- Motivated by a natural problem or an application (VLSI design)
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- A clean abstraction (drawing rules of Shiloach)
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- A systematic explanation of previous results (H-trees)
- A new result (layouts for butterfly & MOT)
- Bonus: geometry...